



本試題共 7 題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (15%) Please solve for $y = y(x)$.

- (a) (05%) $xy' - 3y = 0$
- (b) (05%) $y'' - 4y' + 4y = 0$
- (c) (05%) $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$

2. (15%) Find the integration factor and solution of the ODE equation

$$(x^2 - 4y)dx - xdy = 0$$

3. (10%) Solve ODE solution of $y'' - 4y' + 3y = \sin 2x$

4. (10%) Laplace equation:

- (a) (05%) If $f(t) = \cos(2t + \theta_0)$, $t \geq 0$, please find $L[f(t)]$

- (b) (05%) $F(S) = \frac{1}{(S+1)(S+2)}$, please find $L^{-1}[F(S)]$.

5. (15%) A transform $T(\mathbf{x}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$ and \cdot denotes the inner product.

- (a) (10%) Prove $T(\mathbf{x})$ is linear.

- (b) (05%) If $T(\mathbf{x}) = \mathbf{Ax}$, find the matrix \mathbf{A} .

6. (15%) Let $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ in } \mathbb{R}^3 : x + 2y - z = 0 \right\}$.

- (a) (10%) Prove that W is a subspace of \mathbb{R}^3 .

- (b) (05%) Find the orthogonal complement of W , denoted as W^\perp .

7. (20%) A linear transformation $T(\mathbf{x}) = \mathbf{Ax}$, where

$$\mathbf{A} = \begin{bmatrix} -1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

- (a) (08%) Find the eigenvalues of \mathbf{A} .

- (b) (08%) Suppose $V = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms the eigenbasis for \mathbf{A} , find V .

- (c) (04%) Find the transformation matrix of $T(\mathbf{x})$ when using V from (b) as the basis.



本試題共 5 題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. Beginning with $V_{D,\text{on}} \approx 800\text{mV}$ for each diode, calculate the change in V_{out} if I_{in} changes from 2.0 mA to 2.1 mA in the circuit of Fig. P1 (a), (b), (c). (the thermal voltage, $V_T = 26\text{mV}$)

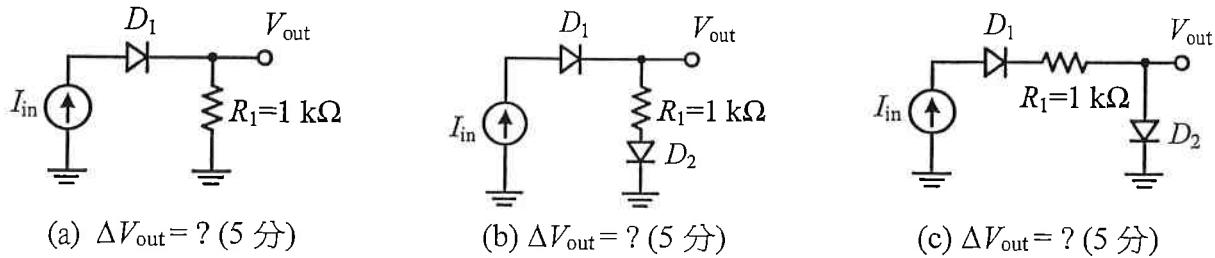


Fig. P1

2. Determine the voltage gain of the stages shown in Fig. P2 (a), (b), (c).

- (a) (5 分) find the voltage gain, $v_{\text{out}} / v_{\text{in}}$ (Assume $\lambda = 0$).
 (b) (10 分) find the voltage gain, $v_{\text{out}} / v_{\text{in}}$ (Assume $\lambda \neq 0$).
 (c) (10 分) find the voltage gain, $v_{\text{out}} / v_{\text{in}}$ (Assume $\lambda \neq 0$).

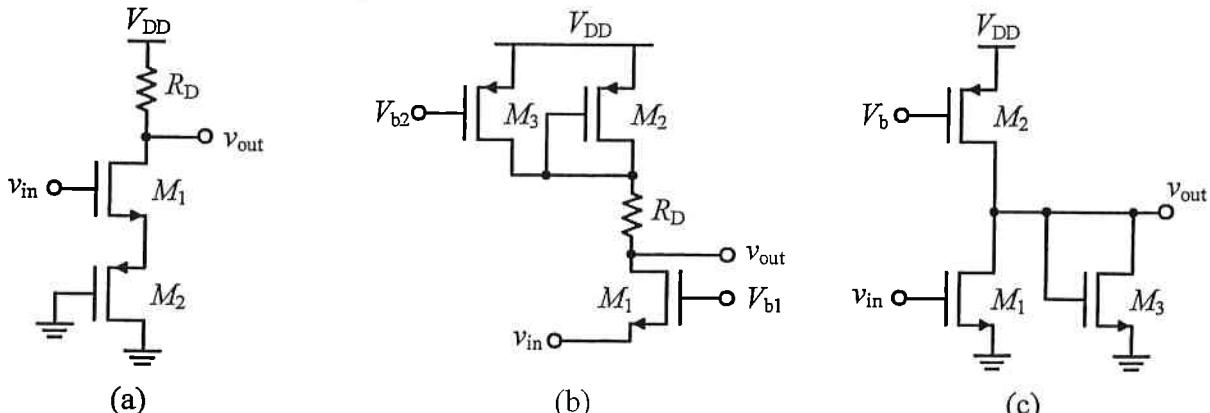


Fig. P2

3. (10 分) Suppose the bipolar transistor in Fig. P3 exhibits the following hypothetical characteristic:

$$I_C = I_S \exp \frac{V_{BE}}{2V_T}$$

and no Early effect. Compute the voltage gain for a bias current of 1 mA.
 (the thermal voltage, $V_T = 26\text{mV}$)

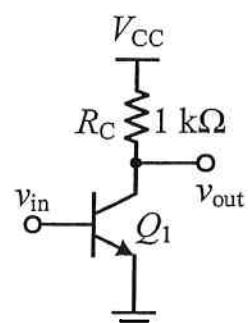


Fig.P3



4. A closed-loop block diagram is shown in Fig. P4.

- (a) (10 分) Derive the transfer function of the loop ($v_{\text{out}} / v_{\text{in}}$).
- (b) (10 分) If A_V is 80 dB and $K_F = 0.5$, what is the total loop gain of this closed loop?

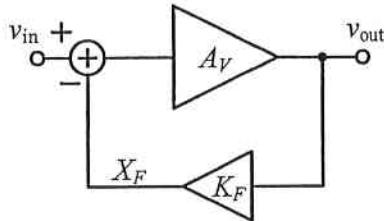


Fig. P4

5. It is known that the common source circuit shown in Fig. P5 has been working in the linear amplification region. (Assume: $V_{\text{DD}} = 10 \text{ V}$, $R_D = 10 \text{ k}\Omega$, $R_S = 100 \Omega$, $C_{GS} = 10 \text{ pF}$, $C_{GD} = 2 \text{ pF}$, $C_{D} = 5 \text{ pF}$, and $g_{m1} = 1 \text{ mA/V}$, M_1 without channel length modulation effect)

- (a) (10 分) Draw the small signal model of this circuit.
- (b) (10 分) Find the pole frequency at v_{in} .
- (c) (10 分) Find the pole frequency at v_{out} .

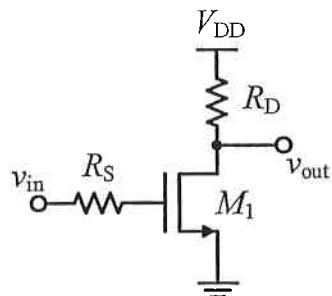


Fig. P5



本試題共 8 題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

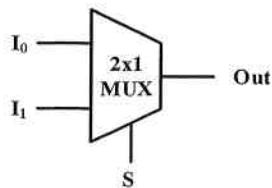
1. (10 pt.) Add the following bit patterns. Leave your results in binary form.

- (a) (2 pt.) $1011 + 0001$
- (b) (2 pt.) $0000 + 1010$
- (c) (2 pt.) $1100 + 0011$
- (d) (2 pt.) $0101 + 0110$
- (e) (2 pt.) $1111 + 0001$

2. (10 pt.) Convert the following unsigned binary numbers to hexadecimal.

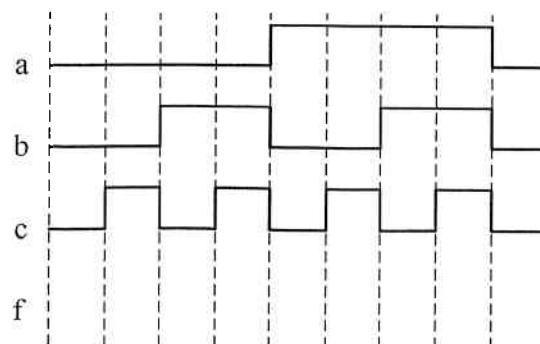
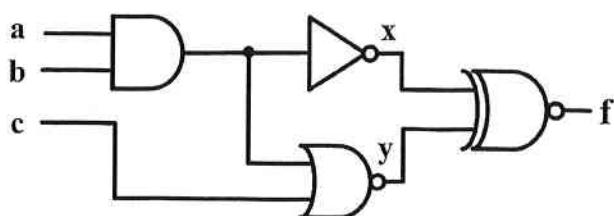
- (a) (2.5 pt.) $1101\ 0001\ 1010\ 1111$
- (b) (2.5 pt.) $001\ 1111$
- (c) (2.5 pt.) 1
- (d) (2.5 pt.) $1110\ 1101\ 1011\ 0010$

3. (10 pt.) A symbol of 2-to-1 mux is listed below. Please draw the gate-level circuit of 2-to-1 mux.



4. (20 pt.) Complete the timing diagram of the following circuit:

Please draw the waveform of the output f.





5. (10 pt.) Assume that the scheduling of a processor is shown in the following table. There are three procedures to be done. Priority Scheduling (PS) scheme for procedure arrangements has been adopted. The priority of procedure P2 is the highest, followed by procedure P3. The lowest priority is the procedure P2. The execution time of the procedures 1-3 are 7, 5, and 4 milliseconds (ms), respectively. What is the average waiting time for a processor?

| Procedures | Orders of Priority | Execution time(ms) |
|------------|--------------------|--------------------|
| P1 | 3 | 7 |
| P2 | 1 | 5 |
| P3 | 2 | 4 |

6. (10 pt.) According to the IEEE 754 single-precision number representation, the binary value is $11000010001010101000000000000000_2$. What is the decimal value?

7. (20 pt.) Emile and William are important company developers. Every time they send a message, they use the Diffie-Hellman (DH) algorithm to generate a set of keys, which will be XOR-encrypted with the message before sending it to each other. Even if others see it, they cannot get the content of the message. According to the DH algorithm, Emile and William selected two public numbers $g=5$ and $p=3$, and each selected two numbers $a=2$ and $b=3$, which are known only to each other. After calculating the key, the ciphertext sent is $0x00_{(16)}$, $0x00_{(16)}$, $0x03_{(16)}$, $06_{(16)}$. What should the plaintext (decoded text) be?

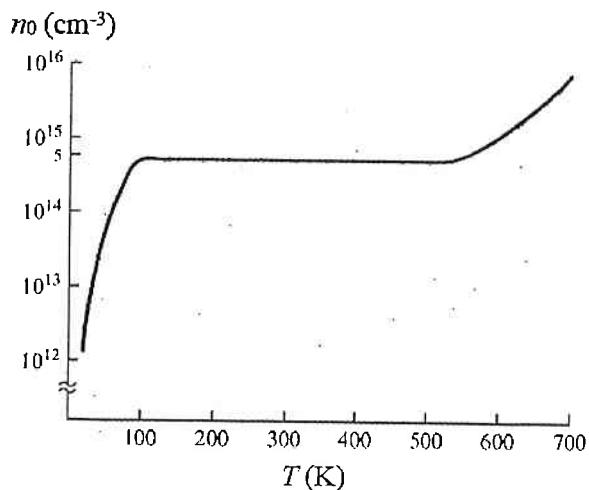
8. (10 pt.) Computers and microprocessors usually contain

1. DDR5 main memory,
2. Register register,
3. Cache
4. SSD high-speed hard disk,

Please arrange from fastest to slowest.



1. 解釋以下名詞：
 - (a) 過量載子(excess carriers) (5%)
 - (b) 多數載子(majority carrier) (5%)
2. 關於能隙，
 (a) 從能帶的角度說明直接能隙(direct bandgap)半導體與間接能隙(indirect bandgap)半導體的差異。(6%)
 (b) 以上哪一種適合做為光電元件的應用？為什麼？(4%)
3. 假設有一塊矽半導體，摻雜了濃度為 $5 \times 10^{14} \text{ cm}^{-3}$ 的磷(P)。摻雜後，半導體的電子濃度隨著溫度的變化顯示如右圖。請說明電子濃度為什麼會呈現如圖的變化趨勢。(10%)
4. 漂移(drift)與擴散(diffusion)是載子在半導體中的二種傳輸機制。請簡單解釋之。(10%)
5. 假設有一塊矽半導體，在 $T = 300 \text{ K}$ 時摻雜之受體(acceptor)濃度為 $N_a = 10^{16} \text{ cm}^{-3}$ 、施體(donor)濃度為 $N_d = 9 \times 10^{15} \text{ cm}^{-3}$ 。假設本質載子(intrinsic carrier)濃度為 $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ 。計算熱平衡之電子濃度與電洞濃度。(10%)
6. 請畫出一 P-N 接面，於平衡狀態下之能帶圖，須包含導帶最下緣(E_C)、價帶最上緣(E_V)、本質費米能階(E_i)與費米能階(E_F)。(15%)
7. 承上，請畫出施加正向偏壓(forward bias)情況下，P-N 接面之能帶圖，並於能帶圖中畫出電子之流動方向。(10%)
8. 請說明何謂費米能階(費米能量 E_F)。(5%)
9. 若一 N 型半導體與一金屬形成蕭特基接觸(Schottky contact)，請畫出於平衡狀態下之能帶圖，需畫出導帶最下緣(E_C)、價帶最上緣(E_V)與費米能階(E_F)。(10%)
10. 請畫出金氧半場效電晶體(MOSFET)之元件結構，並標示出摻雜區域、閘極、汲極與源極。(10%)





Useful physical constants: $\varepsilon_0 \approx \frac{10^{-9}}{36\pi}$ (F/m); $\mu_0 = 4\pi \times 10^{-7}$ (H/m)

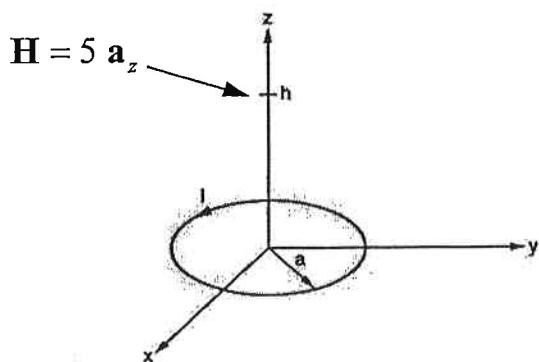
$$\nabla \times \mathbf{H}_{\text{cyl}} = \left[\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right] \mathbf{a}_\rho + \left[\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right] \mathbf{a}_\phi + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho H_\phi) - \frac{\partial H_\rho}{\partial \phi} \right] \mathbf{a}_z$$

$$\nabla \times \mathbf{H}_{\text{spher}} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (H_\phi \sin \theta) - \frac{\partial H_\theta}{\partial \phi} \right] \mathbf{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r} (r H_\phi) \right] \mathbf{a}_\theta + \frac{1}{r} \left[\frac{\partial}{\partial r} (r H_\theta) - \frac{\partial H_r}{\partial \theta} \right] \mathbf{a}_\phi$$

1. 請寫下完整 4 個 Maxwell's equations 的微分式與積分式，並其物理意義。(12 分)
2. 有兩個實心金屬球，半徑分別為 A 球 4 公分，B 球 8 公分；其上所帶電荷量分別對應為 A 球 24-nC 與 B 球 96-nC。當此兩個 A、B 金屬球用一理想金屬導線連接在一起，達平衡時，請說明電荷流動的原因及流動方向？(8 分)
3. 有一理想的平板電容，兩端的電位差為 V_0 ，相距 d ，中間的介電質其介電常數為 ε_r 。依靜電理論，在正電極端的金屬與介電質的界面會個別存有自由移動電荷密度 σ_f 與束縛電荷密度 σ_b 。請寫出靜電學中 **D**、**E**、**P** 三者之構成關係式(Constitutive relation)，並用上述 V_0 、 d 、 ε_r 等符號計算出 σ_f 、 σ_b 。(15 分)
4. 有一柱形界面。半徑 $\rho \leq 2 \text{ cm}$ ，介電質 1 之 $\varepsilon_{r1} = 2$ ； $\rho > 2 \text{ cm}$ ，介電質 2 之 $\varepsilon_{r2} = 4$ 。在柱形界面無累積自由移動電荷，且在界面之介電質 1 側， $\mathbf{E}_1 = 4 \mathbf{a}_\rho + 6 \mathbf{a}_\phi + 9 \mathbf{a}_z$ ；求在界面介電質 2 側之 \mathbf{E}_2 。(5 分)
5. 有一同軸電纜線長 L ，中間金屬軸半徑為 ρ_1 ，外層為一同軸薄金屬層，半徑為 ρ_2 ，中間的非理想絕緣介電質材料其介電常數 $\varepsilon_r = 4.0$ ，電導率 $\sigma = 10^{-3} \text{ S/m}$ 。若此同軸電纜兩端施加一電位差 V_0 ，請推導出此同軸電纜儲存之電能 W_E 與消耗之功率 P 。(10 分)



6. 假設圖中 $(0, 0, h)$ 位置的磁場為 $\mathbf{H} = 5 \mathbf{a}_z$ (A/m)，請問在 $(0, 0, -h)$ 位置的磁場強度是多少？(答案需包括大小、方向、單位) (15 分)



7. 假設磁場為 $\mathbf{H} = (z \cdot \sin\phi) \mathbf{a}_\rho - (\rho^2) \mathbf{a}_z$ (A/m)，請計算在 $(4 \text{ m}, 0^\circ, 4 \text{ m})$ 位置的電流密度 \mathbf{J} 。 (20 分)
8. 假設一磁性材料之磁化率 $\chi_m = 49$ ，而磁通量密度 $B = 0.02 \text{ T}$ 。請計算以下三個值：(答案可用分數表示。若有單位，需寫出單位)
- (1) 相對磁導率 μ_r ；(5 分)
 - (2) 磁場強度 H ；(5 分)
 - (3) 磁化強度 M (magnetization，又稱磁化向量)。(5 分)