



本試題共十題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (5%；複選全對才給分) A and B are 3X3 matrices and $|A| = -3$, $|B| = 2$. Which statements are correct?

(a) $|AB| = -6$; (b) $|2AB^{-1}| = -6$; (c) $|(A^2)^1| = -9$; (d) $|(A^1)^2| = 9$; (e) $|(A^2B^{-1})^1| = -18$

2. (10%) Consider the two vectors, (1, 2, -1) and (3, 1, 0). (a)(2%) Find the norms of the two vectors. (b) (2%) Normalize the two vectors. (c) (6%) Find a vector that is orthogonal to the two vectors.

3. (15%) Consider the matrix $A = \begin{bmatrix} 9 & -3 & 3 \\ -3 & 6 & -6 \\ 3 & -6 & 6 \end{bmatrix}$.

(a) (5%) Find its eigenvalues. (b) (5%) Find the corresponding normalized eigenvectors.

(c) (5%) Find the matrix A^{10} .

4. (10%) Asus and Acer are competing for customers at notebook market. A study has been made of customer satisfaction with the various companies. The results are expressed by the following matrix R. The First column of R implies that 75% of those currently using Asus notebook are satisfied and intend to use Asus next time, while 25% of those using Asus are dissatisfied and plan to use Acer next time. There is a similar interpretation to the second column of R. If the current trends continue, how will the customer distribution eventually settle?

(from)

$$R = \begin{array}{cc} \begin{matrix} \text{Asus} & \text{Acer} \end{matrix} \\ \begin{bmatrix} 75\% & 20\% \\ 25\% & 80\% \end{bmatrix} \begin{matrix} \text{Asus} \\ \text{Acer} \end{matrix} \end{array}$$

5. (5%) Determine the inverse of the matrix $\begin{bmatrix} 5 & 2 & 4 \\ 2 & 1 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, if it exists, using the method of

Gauss-Jordan elimination.

6. (5%) Determine the equation of the polynomial of degree two whose graph passes through the point (1, 6), (2, 3), (3, 2)



7. (15%) Determine the inverse of each of the following matrices, if it exists, using the method of Gauss-Jordan elimination.

(a) (5%)
$$\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$$

(b) (5%)
$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -3 \\ 1 & -2 & 0 \end{bmatrix}$$

(c) (5%)
$$\begin{bmatrix} -3 & -1 & 1 & -2 \\ -1 & 3 & 2 & 1 \\ 1 & 2 & 3 & -1 \\ -2 & 1 & -1 & -3 \end{bmatrix}$$

8. (10%) Solve the following problems.

(a) (5%) Find x such that
$$\begin{bmatrix} 2x & 7 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}.$$

(b) (5%) Find A such that $(4A^t)^{-1} = \begin{bmatrix} 2 & 3 \\ -4 & -4 \end{bmatrix}$, where the superscript t denotes the transpose operation.

9. (9%) Prove that the transformation $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ defined by $T(x, y) = (3x, x + y)$ is linear.

Find the images of the vectors $(1, 3)$ and $(-1, 2)$ under this transformation.

10. (16%) Consider the linear transformation T defined by each of the following matrices.

Determine the kernel and range of each transformation. Show that $\dim \ker(T) + \dim \text{range}(T) = \dim \text{domain}(T)$ for each transformation. (Note that the abbreviations of \dim and \ker denote dimension and kernel, respectively.)

(a) (8%)
$$\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$$

(b) (8%)
$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 3 \\ 2 & 1 & 7 \end{bmatrix}$$



1. (10%) Write a C/C++ function that reads in N integer quiz grades and computes the average and

standard deviation of the N scores. The standard deviation is defined as $\sqrt{\frac{1}{N} \sum_{i=1}^N (s_i - \bar{s})^2}$, where

\bar{s} is average of the N scores and s_i is the i -th score.

2. (10%) Consider the following type definition:

```
struct ShoeType
{
    char style;
    double price;
}
```

Given the function definition corresponding to the following function declarations:

(a) void readShoeRecord(ShoeType& newShoe);

// Fills newShoe with values read from the keyboard.

(b) ShoeType discount(ShoeType oldRecord);

// Returns a structure that is the same as its argument, but with the price reduced by 10%.

3. (5%) What is the output of the following program?

```
#include <iostream>
using namespace std;
void yuntech(int& x, int y, int& z);
int main ( )
{
    int a = 92, b=9 ,c=21;
    yuntech(a,b,c);
    cout<< a <<" " << b << " " << c <<endl;
    return 0;
}
void yuntech(int& x, int y, int& z)
{
    cout<< x <<" " << y << " " << z <<endl;
    x = x-3;
    y = y-3;
    z = z+5;
    cout<< x <<" " << y << " " << z <<endl;
}
```

4. (15%) Given the sequence: 6, 4, 3, 9, 2, 1, 8, 5, 7

(a) (3%) Construct a binary search tree for the sequence.

(b) (3%) Traverse the constructed binary search tree in inorder.

(c) (3%) Construct an AVL tree for the original sequence.



(d) (3%) Construct a heap tree (the root has the maximum key) for the original sequence.

(e) (3%) Construct a 2-3 tree for the original sequence.

5. (5%) What is the output of the following program?

```
#include <iostream>
using namespace std;
main()
{
    const int N=2, M=4;
    int i, j, a[N][M], *p, *q;
    p=&a[0][0];
    q=p+M;
    for (i=0; i<M; i++)
    {
        *(p+i)=N+i;
        *(q+i)=*(p+i)+i;
    }
    for (i=0; i<N; i++)
    {
        for (j=0; j<M; j++)
        {
            cout<< a[i][j]<<" ";
        }
        cout<<endl;
    }
}
```

6. (5%) What is the output of the following program?

```
#include <iostream>
using namespace std;
int csie (int n) {
    if (n<2)
        return 2;
    return csie(n-1)-csie(n-2);
}
main () {
    int i;
    for (i=0; i<7; i++)
        cout<< i << csie(i)<<endl;
}
```



7. (10 %) If the address of array elements $A(1,1)$ and $A(3, 3)$ are 2204 and 2244, what is the address of $A(4,4)$
8. (10 %) A byte of data with binary representation is 10011010. Please derive its hamming code.
9. (10 %) Please write the prefix and postfix notations of $A+B*(C-D)/E$
10. (10 %) Please implement the following function F with a multiplexer

$$F = A'B'C + A'BC + AB'C + ABC'$$
11. (10 %) Based on the Fig. 1, please write the search sequence with breadth-first search and depth-first search, respectively.

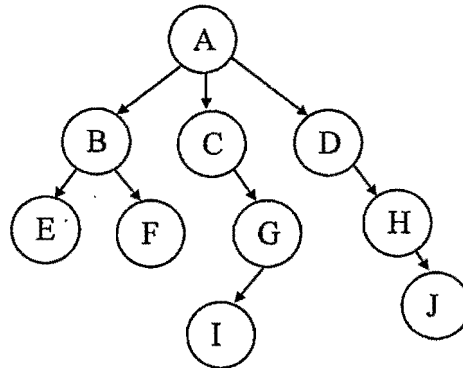


Fig.1



1. In problems (a)-(c), please solve for $y = y(x)$ (15 分)

(a) $y'' - 12y = 0$

(b) $y'' - \frac{2}{x+1}y' + \frac{2}{(x+1)^2}y = 0$

(c) $xy'' + (1-2x)y' + (x-1)y = 0$

2. A function $y = y(x)$ is the 1st order differential equation: (10 分)

$$\frac{dy}{dx} = -\frac{3x^2y + 6xy + y^2}{3x^2 + y}$$

(a) Does the differential equation satisfy the "Condition of Exactness"?

(b) Solve the differential equation using the method of integration factor.

3. A Bernoulli's differential equation $\frac{dy}{dx} + p(x)y = g(x)y^a$ have the value of a is any real number but not equal to 0 or 1. (15 分)

(a) Set $u(x) = [y(x)]^{1-a}$ and show that the above differential equation can be transformed into a linear form. (5 分)

(b) Use the result of (a) to solve: $\frac{dy}{dx} + \frac{y}{x} = -2xy^2$ (10 分)

4. Use Laplace transforms to solve the equation system in the initial condition of $x(0) = 2, y(0) = 0$. (10 分)

$$\begin{cases} x' + 3x - y = 2 \\ x' + y' + 3x = 0 \end{cases}$$

5. Find the inverse of $A = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{pmatrix}$. (15 分)



6. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix}$. (15 分)
7. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6i+2j+3k$. (10 分)
8. If $F = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$, find (a) curl F (b) div F (c) div (curl F). (10 分)



1. Both the basic current mirror and cascode current mirror are shown in Fig. 1, please answer following questions:

(a) (5%) Point out the reason of current mismatch in the basic current mirror and express the output current I_o of the basic current mirror in terms of I_{REF} .

(b) (5%) State the reason that the cascode current mirror have an advantage over the basic current mirror.

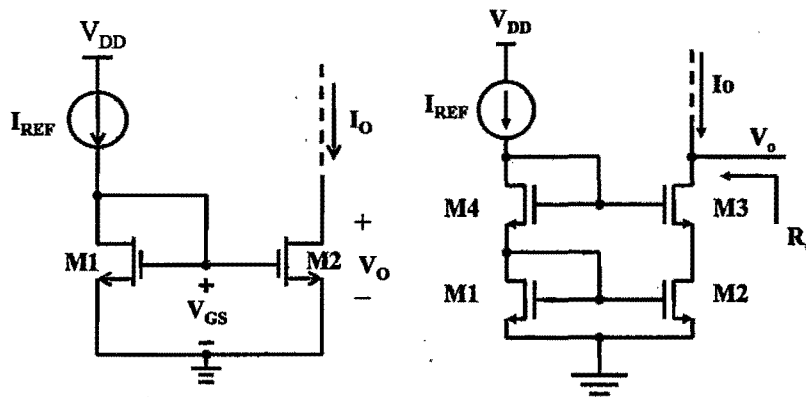


Fig. 1

2. The amplifier shown in Fig. 2 has $R_{sig} = R_L = 1\text{ k}\Omega$, $R_C = 1\text{ k}\Omega$, $R_B = 47\text{ k}\Omega$, $\beta = 100$, $C_\mu = 0.8\text{ pF}$, and $f_T = 600\text{ MHz}$.

(a) (5%) Find the dc collector current of the transistor.

(b) (5%) Find g_m and r_π .

(c) (5%) Find the midband voltage gain from base to collector (Neglect the effect of r_o and R_B) and use the gain to find the R_{in} .

(d) (5%) Find C_{in} .

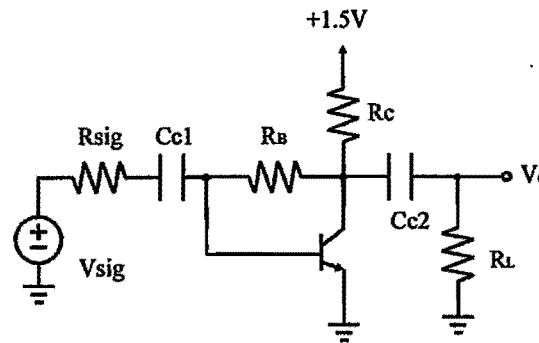


Fig. 2

3. An amplifier having a low-frequency gain of 10^3 and poles at 10^4 Hz and 10^5 Hz is operated in a closed negative-feedback loop with a frequency-independent β .
- (5%) For what value of β do the closed-loop poles become coincident, i.e. pole 1 equal to pole 2? And at what frequency?
 - (5%) What is the low-frequency gain corresponding to the situation in (a)? What is the value of the closed-loop gain at the frequency of the coincident poles?
 - (5%) What is the value of quality factor Q corresponding to the situation in (a)?
 - (5%) If β is increased by a factor 10, what are the new pole locations?



4. (20%) (a) If A is an ideal amplifier, $\frac{V_{out}}{V_{in}} = ?$ (b) If $A(s) = \frac{A_0}{1 + \frac{s}{\omega_p}}$, where $A_0 = 80\text{dB}$ and $\omega_p = 2\pi \times 100 \text{ rad/sec}$, what is the 3dB frequency of the closed-loop amplifier?

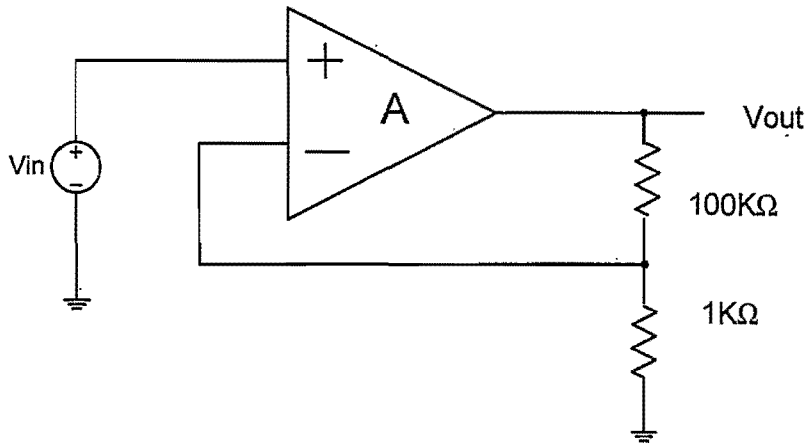


Fig. P4

5. (10%) If the effect of channel-length modulation is negligible, calculate the low-frequency small-signal voltage gain. Note that $\mu_n = 4\mu_p$ and $(\frac{W}{L})_n = 4(\frac{W}{L})_p$.

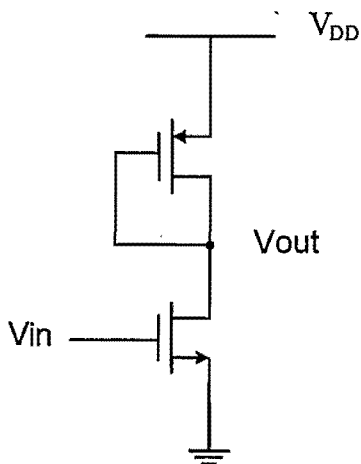


Fig. P5



6. (20%) If the operational amplifier A is ideal, write down (a) the differential gain

$\frac{V_{out}}{V_2 - V_1} = ?$ (b) the input resistance of the differential amplifier.

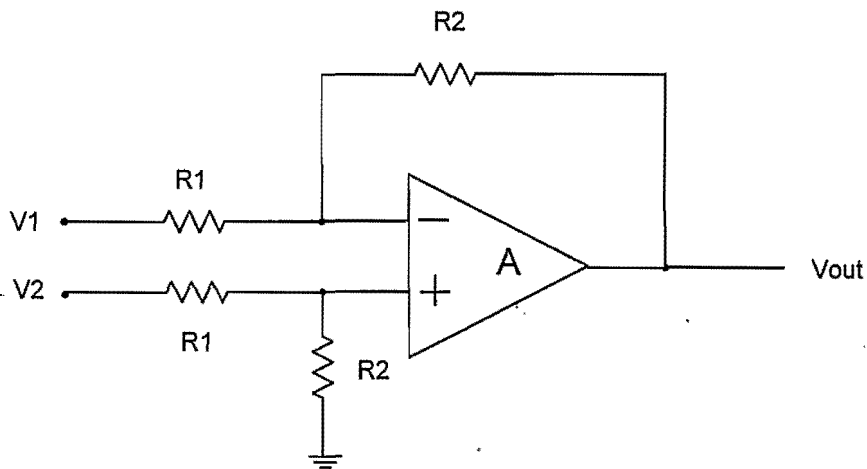


Fig. P6



總分 100 分 共 6 題

1. Selection problem set:

[16%]

(1) Following experimental and physic phenomena may prove particle with wave properties.

(A) SEM (B) tunneling (C) hydrogen spectra (D) photoelectrical effect

(2) When temperature is increased, the Fermi-level of p-type semiconductor is

(A) closed to E_V (B) far from E_C (C) closed to E_{Fi} (D) unchanged

(3) In Hall-measurement, the magnetic field is upper ward and the current is followed from left to right. The frond of a semiconductor is measured positively biased. This semiconductor is

(A) n-type (B) p-type (C) intrinsic (D) undecided

(4) The n-type and p-type semiconductors show the same doping concentration which shows smaller resistance?

(A) n-type (B) p-type (C) the same (D) undecided

2. How make metal-semiconductor be an Ohmic contact?

[16%]

3. Explain (a) the quasi-Fermi level, (b) effective mass of electron in material, and (c) ionized impurity scattering

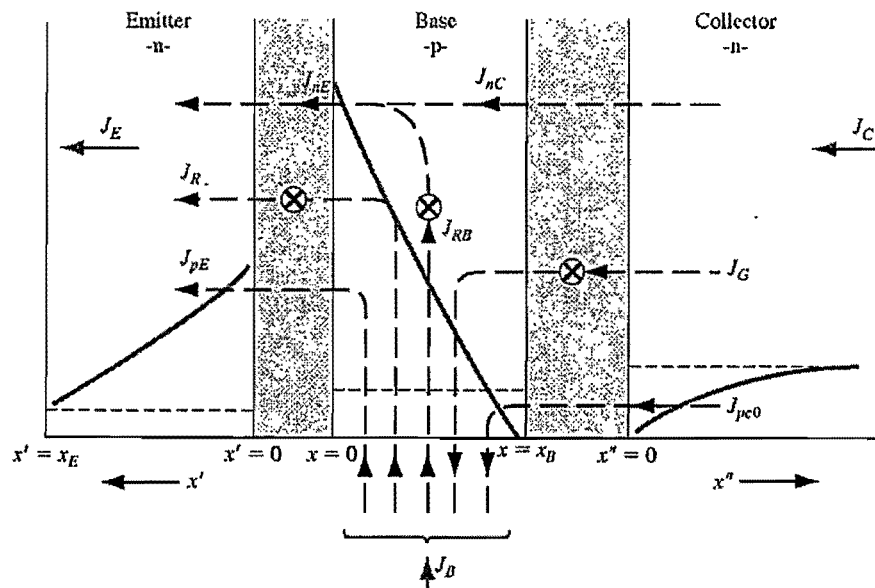
[18%]

4. Explain two physical mechanisms of the reverse-bias breakdown in a pn junction.

[15%]

5. The current components of an npn bipolar transistor in forward-active mode is shown in the figure. Write down the definition of "Emitter Injection Efficiency Factor," and explain how to improve it.

[20%]

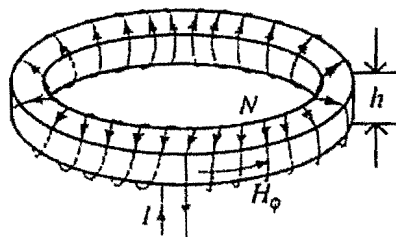


6. For an n-channel enhancement-mode MOSFET and an n-channel depletion-mode MOSFET, (a) explain their difference in the channels, (b) explain their difference in the threshold voltages.

[15%]



- Given a scalar field $V = 2xy - yz - xz$,
 - find the vector representing the direction and the magnitude of the maximum rate of increase of V at point $P(2, -1, 0)$, and (5%)
 - find the rate of increase of V at point P in the direction toward the point $Q(0, 2, 6)$. (5%)
 - If this scalar field V represents some electrical potential, determine the electric field intensity \mathbf{E} . (5%)
- An inhomogeneous dielectric fills a parallel-plate capacitor of surface area A and thickness d . By measuring from the bottom plate, the dielectric constant is $\epsilon_r = (1 + z)$.
 - Calculate the capacitance. (10%)
 - Calculate the electrostatic potential energy stored in this capacitor if a 9.0-V potential is applied across the conductors. (5%)
- A block of iron (99.8% pure, $\mu_r = 5000$) exists for $z < 0$. For $z > 0$, we have air and a magnetic flux density $\mathbf{B}_{\text{air}} = 1\mathbf{a}_x + 4\mathbf{a}_y + 12\mathbf{a}_z$ T. Assuming there is no sheet current at the interface, find \mathbf{B}_{iron} . (15%)
- Determine the electric field \mathbf{E} at $(8, 0, 0)$ m due to a charge of 10 nC distributed uniformly along the x axis between $x = -5$ m and $x = 5$ m. (15%)
- An electron and photon separated by a distance of 10^{-11} m are symmetrically arranged along the z axis with $z = 0$. Find (a) the dipole moment, (b) the potential and (c) the electrical field at $(3, 4, 12)$. (15%)
- A toroidal winding with N turns, as shown in the figure, has inner radius a , the outer radius b and the height of the ring h . What is (a) the magnetic field intensity within the ring, (b) the energy stored in the magnetic field of the toroidal winding (if the winding carries a current of I amperes)? (15%)



- Write down the Maxwell's equations (differential form) and the physical meanings. (10%)