



本試題共七題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. (20%) Consider the matrix

$$A = \begin{bmatrix} 2 & -2 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

- (a) (5%) Find its characteristic polynomial.  
 (b) (5%) Find its eigenvalues.  
 (c) (10%) Find the corresponding normalized eigenvectors.

2. (10%) Determine the rank of

$$A = \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -9 \\ 1 & 2 & -10 \\ -3 & 4 & -60 \\ 7 & 8 & -16 \\ -6 & 4 & -84 \end{bmatrix}.$$

3. (10%) Prove that  $(AB)^T = B^T A^T$ , where  $A$  and  $B$  are matrices, and  $T$  represents the transpose operation.

4. (10%) Prove that a square matrix  $A$  is invertible if and only if  $\det(A) \neq 0$ .

5. (15%) Find a basis for the column space of the following matrix  $A$ .

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \\ -1 & -4 & 6 \end{bmatrix}$$

6. (20%) Consider the linear transformation  $T(x, y) = (3x + 4y, 5x + 7y)$  of  $\mathbf{R}^2 \rightarrow \mathbf{R}^2$ . Prove that  $T$  is invertible and find the inverse of  $T$ .

7. (15%) Show that the following matrix  $A$  is not diagonalizable.

$$A = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$$



1. Suppose a tank contains 200 gallons of brine (salt mixed with water), in which 100 pounds of salt are dissolved. A mixture consisting of  $\frac{1}{8}$  pound of salt per gallon is flowing into the tank at a rate of 3 gallons per minute, and the mixture is continuously stirred. Meanwhile, brine is allowed to empty out of the tank at the same rate of 3 gallons per minute. How much salt is in the tank at any time? (10%)

2. Given  $6x^2y + 12xy + y^2 + (6x^2 + 2y)y' = 0$

(a) show that the differential equation is not exact .

(b) find an integrating factor .

(c) find the general solution (perhaps implicitly defined). (15%)

3. Find the solution of the equation:

$$x^2y'' - 5xy' + 8y = 2\ln x, \quad x > 0 \quad (10\%)$$

4. Use Laplace Transform technique , find the initial value problem:

$$y'' - 8y' + 16y = 3 ; y(0) = y'(0) = 0. \quad (15\%)$$

5.  $A = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$  Compute (a)  $A^2$  (b)  $A^{60}$  (10 分)

6. Find a unit normal vector  $\bar{n}$  of the cone of revolution  $z^2 = 3(x^2 + y^2)$  at the point

$$P:(1,0,2) \quad (10 \text{ 分})$$

7.  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(a) Find eigenvalues and eigenvectors of Matrix A

(b) Compute  $A^4 + 2A^3 - 20A^2 - 66A - 45I = 0$  (15 分)

8. If  $\bar{F} = 2x^2\bar{i} + 2y^2\bar{j} + 2z^2\bar{k}$  ,  $f = 2x^3 - 2yz^2$ , Find the value at point( 1,1,1) (a)

$$\bar{\nabla} \cdot (\bar{\nabla} f) \quad (b) \quad \bar{\nabla} \cdot (f\bar{F}) \quad (c) \quad \bar{\nabla}(\bar{\nabla} \cdot \bar{F}) \quad (15 \text{ 分})$$



本試題共九題，每題得分如各題中所示，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. For the circuit in Fig. P1, with ideal diode, find:

- (1) (5 分) the values of  $i_1, i_2, i_L, v_1, v_o,$
- (2) (5 分) the current gain  $i_o/i_1,$
- (3) (2 分) the power gain  $P_o/P_i.$

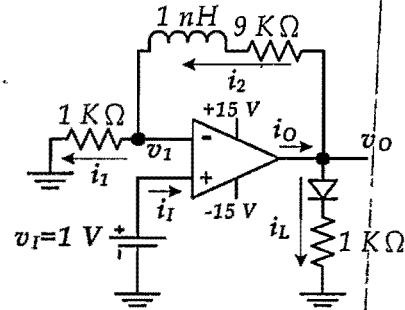


Fig. P1

2. Assume a 0.5 V drop across each conducting diode in Fig. P2. If the magnitude of the average of each output is to be 5 V, find:

- (1) (5 分) the required amplitude of the sine wave across the entire secondary winding.
- (2) (5 分) the peak inverse voltage of each diode.

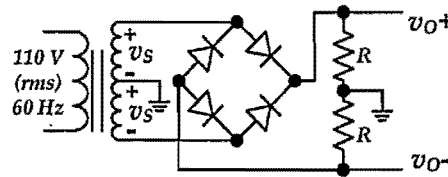


Fig. P2

3. Show the channel current  $I_D$  equations of the  $n$ -channel and  $p$ -channel enhancement-type MOSFET in:

- (1) (4 分) triode region.
- (2) (4 分) saturation region.

(where  $\mu$  = carrier mobility,  $W$  = channel width,  $L$  = channel length,  $C_{ox}$  = unit capacitance,  $V_T$  = threshold voltage, and  $\lambda$  = channel length modulation parameter)

4. Explain the following effect or phenomenon of the MOSFET in detail:

- (1) (5 分) the body effect.
- (2) (5 分) channel-length modulation.

5. (10 分) The voltage  $V_E$  is 1 V in Fig. P5. Under the assumption that  $|V_{BE}| = 0.5$  V, what are  $I_E, V_B, I_B, I_C,$  and  $V_C$ ?

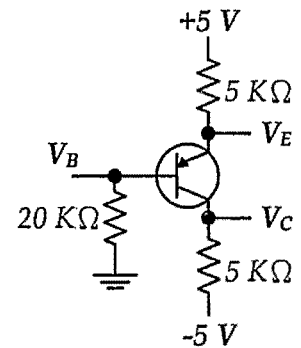


Fig. P5



6. (13 分) Does the cascode MOSFET mirror shown in Fig. P6(a) achieve: (A) a decrease in output resistance (B) an increase in output resistance (C) no effect in output resistance (D) unpredictable effect in output resistance, over the basic MOSFET current mirror shown in Fig. P6(b)? (單選題 — A, B, C, D 選擇一答案)

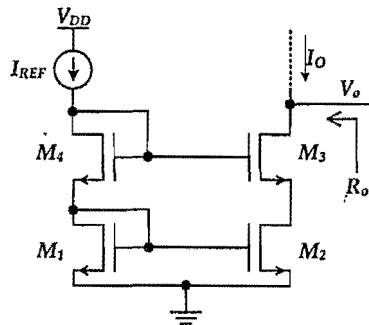


Fig. P6(a)

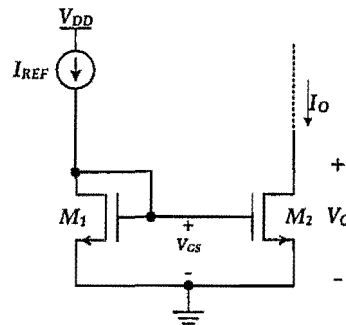


Fig. P6(b)

7. (13 分) Which does the following factor not contribute to the DC offset voltage of the MOS differential pair shown in Fig. P7: (A) mismatch in body-effect parameter (B) mismatch in load resistance (C) mismatch in aspect ratio  $W/L$  (D) mismatch in threshold voltage?

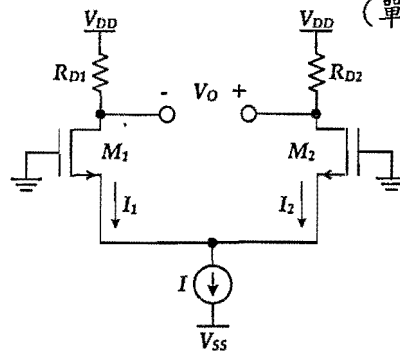


Fig. P7

8. Consider an amplifier having a midband gain  $A_M$  and a low-frequency response characterized by a pole at  $s = -\omega_L$  and a zero at  $s = 0$ . Let the amplifier be connected in a negative-feedback loop with a feedback factor  $\beta$ .

- (1) (6 分) Find an expression for the midband gain of the closed-loop amplifier.
- (2) (6 分) Find an expression for the lower 3-dB frequency of the closed-loop amplifier.

9. Negative feedback is to be used to modify the characteristics of a particular amplifier for various purposes. Identify the feedback topology to be used if:

- (1) (6 分) Input resistance is to be lowered and output resistance raised: (A) series-shunt (B) series-series (C) shunt-shunt (D) shunt-series? (單選題 — A, B, C, D 選擇一答案)
- (2) (6 分) Input resistance is to be raised and output resistance lowered: (A) series-shunt (B) series-series (C) shunt-shunt (D) shunt-series? (單選題 — A, B, C, D 選擇一答案)



1. (10%) Given the structure type and variable definitions.

```

struct ShoeSize
{
    char width;
    int number; };
struct ShoeType
{
    char style;
    ShoeSize size;
    double price; };
ShoeType shoe1, shoe2;
  
```

What type do these variables have?

- (a) shoe1.style
  - (b) shoe2.size
  - (c) shoe1.size.width
  - (d) shoe2.price
  - (e) shoe1.size.number
2. (10%) Suppose your program contains the following class definition (along with definitions of the member functions):

```

class YourClass
{
public:
    YourClass (int newInfo, char moreNewInfo);
    YourClass( );
    void doStuff( );
private:
    int information;
    char moreinformation;
};
  
```

Which of the following are legal?

- (a) YourClass anObject(42, 'A');
  - (b) YourClass anotherObject;
  - (c) YourClass yetAnotherObject( );
  - (d) anObject = YourClass(99, 'B');
  - (e) anObject = YourClass( );
  - (f) anObject = YourClass;
3. (10%) Answer the following questions regarding an array called table:
- (a) (3%) Declare the array to be an integer array and to have 3 rows and 3 columns.
  - (b) (4%) Use a for repetition statement to initialize each element of the array to the sum of its subscripts. Assume that the integer variables i and j are declared as control variables.
  - (c) (3%) Write a program segment to print the values of each element of array table in



tabular format with 3 rows and 3 columns.

4. (5%) What is the output of the following program?

```
#include <iostream >
using namespace std;
void f(int n);
int t=0;
int main (void)
{
    f(9);
    cout << t <<endl; }
void f(int n)
{
    if (n>1)
    {
        t++;
        if (n%2==0)
            f(n/2);
        else
            f(3*n+1); }
}
```

5. (5%) What is the output of the following program?

```
#include <iostream>
using namespace std;
int p=10;
main ( )
{
    int sub1(void);
    void sub2(int);
    void sub3(int*);
    int a = 5, b=8 ,i;
    for (i =1;i<=3;i ++)
    {
        sub2(a);
        sub3(&b);
        cout<< "it " << i << "th pass =" << sub1()<< a << b <<p
        <<endl;    }
    }
int sub1(void)
{
    static int x=0;
    x++;
    return(x); }
void sub2 (int y)
{ y--; p-=y;}
void sub3 (int *z)
{ *z+=1; }
```



6. (5%) Write a definition for a void-function that has two int value parameters and outputs to the screen the product of these arguments. Write a main function that asks the user for these two numbers, reads them in, calls your function, then terminates.

7. (5%) Consider the following function and code segment.

```
void One( int first, int & second )
{
    first = 17;
    second = first + 1; }
int main()
{
    int j = 4;
    int k = 3;
    One(j, k); }
```

After the call to One (j, k) ; what are the values of j and k?

8. (6%) Number system conversion

(a) Binary to octal number :  $(10110001101011.111100000110)_2 = (?)_8$

(b) Hexadecimal to binary number :  $(306.D)_{16} = (?)_2$

9. (6%) Complements

(a) Please find the 1's complement of  $(1011000)_2 = (?)$

(b) Please find the 10's complement of  $(246700)_{10} = (?)$

10. (12%) Postulates and Theorems of Boolean Algebra

(a) DeMorgan  $(x+y)' = ?$

(b) DeMorgan  $(xy)' = ?$

(c) Absorption  $x+xy = ?$

(d) Absorption  $x(x+y) = ?$

11. (5%) Sum of minterm to product of maxterm

$F(A, B, C) = \sum(1, 4, 5, 6, 7) = \prod(?)$

12. (5%) Simplify the Boolean function  $F=A'C+A'B+AB'C+BC$  as sum of products expression?



13. (8%) Simplify the Boolean function

$$F(w,x,y,z) = \Sigma(1,3,7,11,15)$$

which has don't care conditions (terms)

$$d(w,x,y,z) = \Sigma(0,2,5)$$

as sum of products expression?

14. (8%) Please reduce the state diagram

