



1. The transfer function of filter circuit is given by  $H(S) = \frac{20 S}{S^2 + 20 S + 10^4}$ ,

(10%) (a). Find  $\omega_0$ ,  $Q$ , and damping ratio.

(10%) (b). Sketch the gain Bode plot with detailed calculation.

2.

(3%)(a). In the Fig.1, find differential-mode gain,  $A_d$ , if  $R_1 = R_1^*$ ,  $R_2 = R_2^*$ ,  $R_3 = R_3^*$ .

(7%)(b). In the Fig.1, find  $v_o$  if  $R_1 \neq R_1^*$ ,  $R_2 \neq R_2^*$ ,  $R_3 \neq R_3^*$ .

(7%)(c). In the Fig.1, explain that how to optimize common-mode rejection ratio if

$$R_1 \neq R_1^*, R_2 \neq R_2^*, R_3 \neq R_3^* \text{ and } R_3 \text{ is adjustable.}$$

(3%)(d). In the Fig.2, find  $v_o$ .

3. A FET-based Colpitts oscillator is shown in Fig.3. The gate resistance is very large,  $C_G$  and  $C_S$  are very large coupling and bypass capacitors.

(15%)(a) Find the oscillation frequency if the high-frequency small signal model is used.

(5%)(b) Find the started condition of oscillation.

4. A CMOS inverter is shown in Fig.4-1.

(4%)(a). The transfer curve of inverter is shown in Fig.4-2. Try to complete the Table 4-1.

		Operation region		Operation region
I	NMOS		PMOS	
II	NMOS		PMOS	
III	NMOS		PMOS	
IV	NMOS		PMOS	
V	NMOS		PMOS	

Table 4-1

(8%)(b). Assume  $|V_m| = |V_p| = 1V$ ,  $K_n = 4K_p = 100 \frac{\mu A}{V}$  where  $V_m$  and  $V_p$  are threshold

voltage of NMOS and PMOS,  $K_n = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n$  and  $K_p = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p$ . Try

to find  $V_{th}, V_A, V_B$ .



(4%)(c). Apply a 1MHz square wave swept between 0V and 5V to  $V_{in}$  and find average power consumption at  $V_{out}$ .

(4%)(d). In most real case, why  $(\frac{W}{L})_n \neq (\frac{W}{L})_p$  ?

5. A CMOS feedback amplifier is shown in Fig.5. If the DC input voltage is zero, please calculate the following parameters:

(14%)(a) The overall gain  $\frac{v_o}{v_i}$ .

(6%)(b) The output resistance.

Use  $\mu_n C_{ox} = 60 \frac{\mu A}{V^2}$ ,  $\mu_p C_{ox} = 30 \frac{\mu A}{V^2}$ ,  $V_{tn} = 0.8V$ ,  $V_{tp} = -0.8V$ ,  $\lambda = \frac{1}{V_A} = 0.03$ . The

dimensions of M1, M2, M6, M7, M8 are  $\frac{W}{L} = \frac{40}{1}$  and M3, M4, M5 are  $\frac{W}{L} = \frac{20}{1}$ .

( $\sqrt{480000} = 692.82$  and  $\sqrt{960000} = 979.8$ )

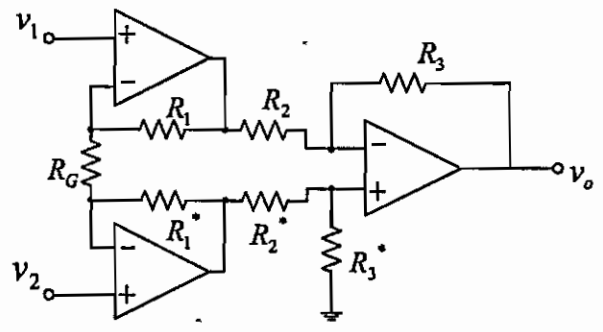


Fig.1

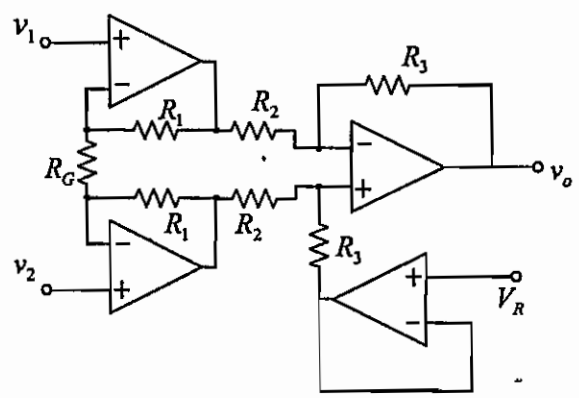


Fig.2

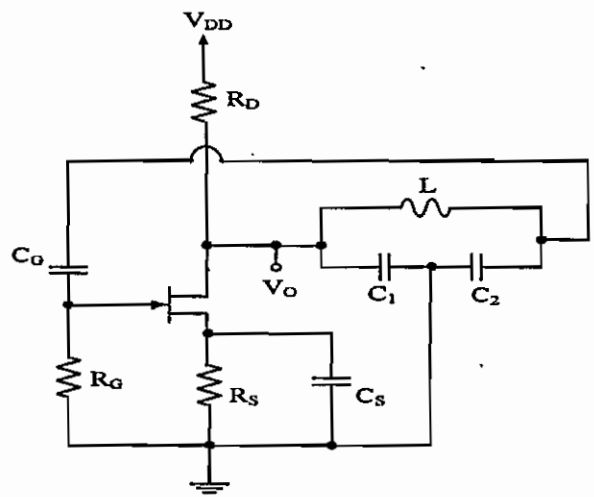


Fig.3

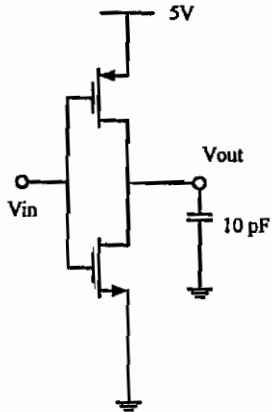


Fig. 4-1

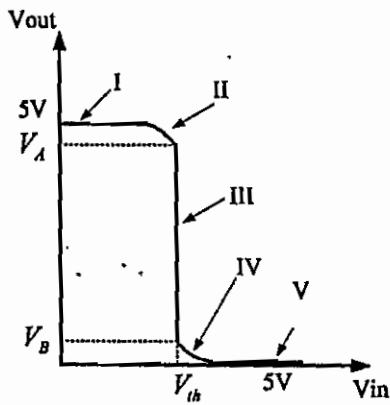


Fig. 4-2

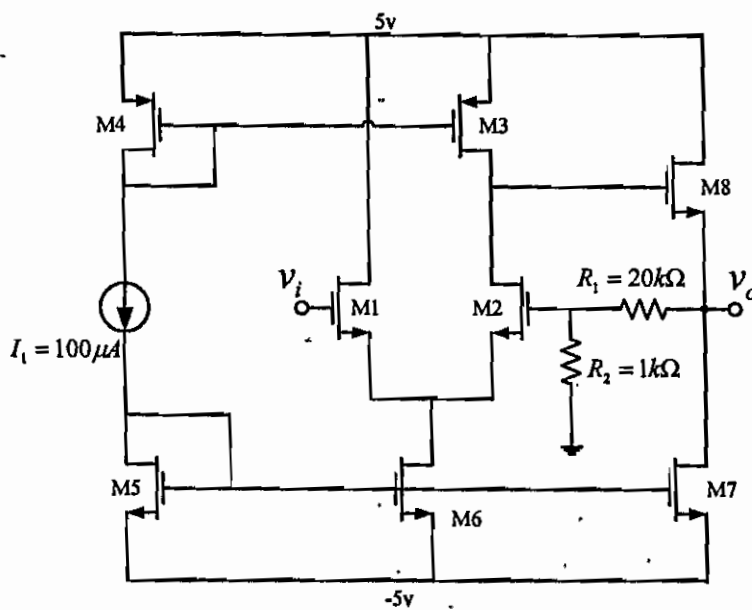


Fig. 5



1. Given the 4-variable K-map as Fig. P1, circulate the 1's and write the minimized expression. (10%)

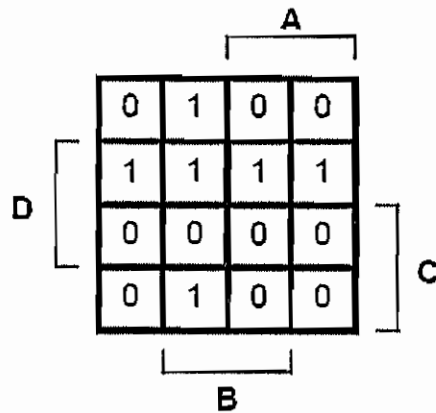


Fig. P1

2. (a) Using only one level of logic, draw the static CMOS circuit schematic for the function:

$$Z = [A(B + C) + DE + F]$$

Draw your circuit in such a way that the number of transistor drains at the output node is minimized. (10%)

- (b) What is the logic gate function in Fig. P2? (5%)

- (c) Draw the static CMOS circuit schematic for the logic gate in Fig. P2. (5%)

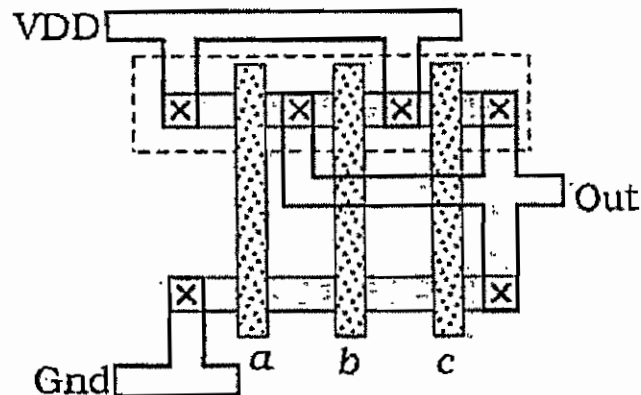


Fig. P2



3. A proposed edge-triggered D-type flip-flop is shown in Fig. P3. (The weak inverter has a weak PMOS and a weak NMOS.)

- (a) Is this a rising-edge triggered or falling-edge triggered flip-flop? (5%)
- (b) Is Z the true or the complement Q output? (5%)
- (c) For the 4 possible combinations (current value of Z, next D to be stored), state what will happen at node X and node Y during the evaluate phase. (10%)

current Z	next D	node X and Y during the evaluate phase

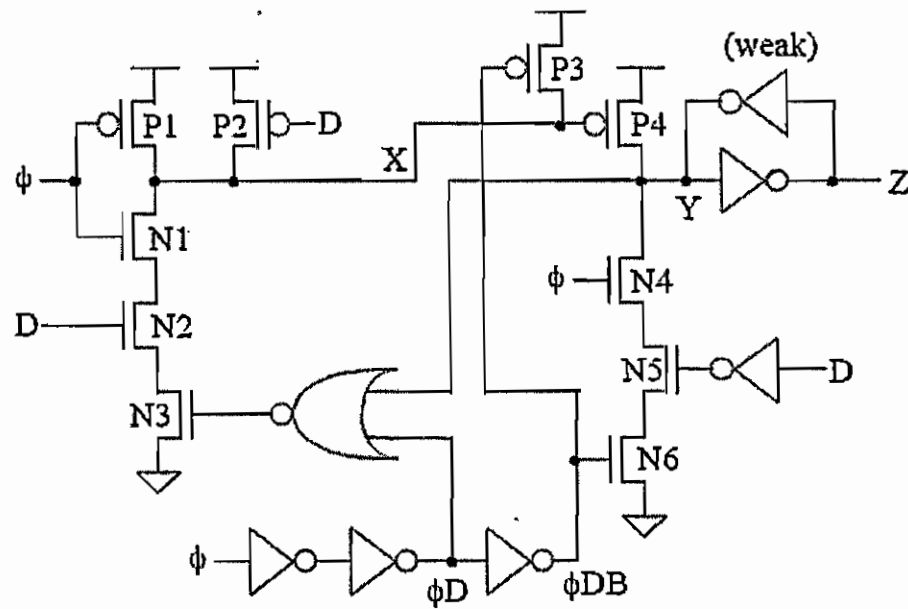


Fig. P3



4. For the amplifier given in Fig. P4, what is the small-signal voltage gain? All transistors are biased in the saturation region. Neglect the channel-length modulation and body effect. (10%)

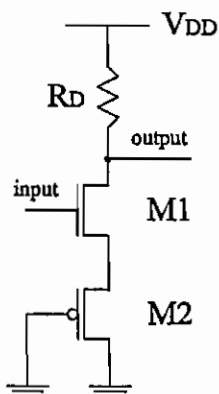


Fig. P4

5. An amplifier is shown in Fig. P5. The NMOS transistor is biased in the saturation region. Neglect the channel-length modulation effect. Find the transresistance gain  $\frac{V_{out}}{i_{in}}$ . (20%)

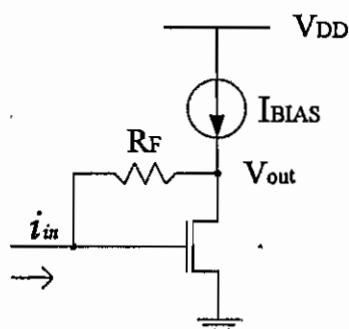


Fig. P5

6. For the circuit given in Fig. P6, M1 is an NMOS transistor and A is an operational amplifier.  $V_{GS} = 0.67V$ ,  $R1 = 10k\Omega$ ,  $R2 = 6k\Omega$ ,  $R3 = 2k\Omega$ . Please calculate  $I_{DS}$  of M1. (20%)

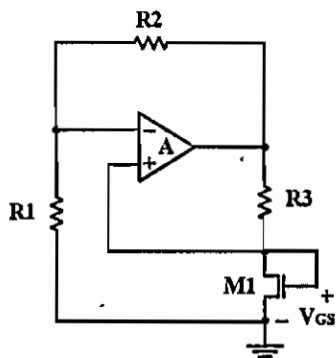


Fig. P6



1. Three vectors  $\vec{b}$ ,  $\vec{v}_1$  and  $\vec{v}_2$  are in  $\mathbb{R}^3$ . (10%)
  - (a) What's the condition that  $\vec{b}$  is called linear combination of the vectors  $\vec{v}_1$  and  $\vec{v}_2$ ? (5%)
  - (b) Is the vector  $[-3 \ 0 \ 3]^T$  a linear combination of  $[1 \ 2 \ 3]^T$  and  $[4 \ 5 \ 6]^T$ ? (5%)
  
2.  $V_i$  are sets in the vectors space  $\mathbb{R}^3$ . (10%)
 
$$V_1 = \{(x_1, x_2, x_3) \mid x_2 \geq 0\},$$

$$V_2 = \{(x_1, x_2, x_3) \mid x_1 = x_2 = 2x_3\},$$
  - (a) State the requirements of a set can be a subspace. (5%)
  - (b) Whether  $V_i$  are subspaces of the vectors space  $\mathbb{R}^3$ . (5%)
  
3. Let  $T$  be a linear transformation from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  that projects any vector orthogonally onto the line  $L$  spanned by the vector  $[4 \ 3]^T$ . (15%)
  - (a) Find the eigenvalues and their corresponding eigenvectors for  $T$ . (10%)
  - (b) By using the eigenvectors you found in (a) as the basis, find a transformation matrix  $A$  for  $T$ . (5%)
  
4. A linear system is as following: (15%)

$$A\vec{x} = \vec{b}, \text{ where } A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \\ 1 & 3 \\ 1 & -1 \end{bmatrix} \text{ and } \vec{b} = \begin{bmatrix} -2 \\ 0 \\ 0 \\ -2 \end{bmatrix}$$

- (a) Find an orthonormal basis  $\{\vec{u}_1, \vec{u}_2\}$  of  $\text{im}(A)$ , where  $\text{im}(A)$  is the image of  $A$ . (5%)
- (b) To find the least-square solution  $\vec{x}^*$  of the system. (5%)
- (c) Graph the geometric relationship among  $A\vec{x}^*$ ,  $\text{im}(A)$ , and  $\vec{b}$ . (5%)





5. The color of light can be represented in a vector  $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ , where  $R$  = amount of red,  $G$  = amount of green, and  $B$  = amount of blue. The human eye and the brain transform the incoming signal into the signal  $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$ , where
- $$\text{intensity } I = \frac{R+G+B}{3} \quad (10\%)$$

$$\text{long-wave signal } L = R - G$$

$$\text{short-wave signal } S = B - \frac{R+G}{2}.$$

- (a) Find the matrix  $P$  representing the transformation from  $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$  to  $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$ . (5%)
- (b) Consider a pair of yellow sunglasses for water sports that cuts out all blue light and passes all red and green light. Find the  $3 \times 3$  matrix  $A$  that represents the transformation incoming light undergoes as it passes through the sunglasses. (5%)
6. Consider the linear transformation  $T(f) = f' + f''$  from  $P_2$  to  $P_2$ , where  $P_2$  is the set of all polynomials of degree  $\leq 2$ . Please find a  $3 \times 3$  matrix  $B$  for this linear transformation  $T$ . Note that  $f'$  and  $f''$  are the first order and second order derivatives of  $f$ , respectively. (10%)
7. Find the derivative of the function (10%)

$$f(x) = \det \begin{bmatrix} 1 & 1 & 2 & 3 & 4 \\ 9 & 0 & 2 & 3 & 4 \\ 9 & 0 & 0 & 3 & 4 \\ x & 1 & 2 & 9 & 1 \\ 7 & 0 & 0 & 0 & 4 \end{bmatrix}.$$

8. Find an orthogonal matrix  $S$  and a diagonal matrix  $D$  such that  $S^{-1}AS = D$ . (10%)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$

9. Let  $L$  be a lower triangular  $3 \times 3$  matrix with positive entries on the diagonal. Please find  $L$

$$\text{such that } A = LL^T, \text{ where } A = \begin{bmatrix} 4 & -4 & 8 \\ -4 & 13 & 1 \\ 8 & 1 & 26 \end{bmatrix}. \quad (10\%)$$