

系所:機械系

科目:工程數學(1)

Prob. 1 (25%)

Consider the ordinary differential equation (O.D.E.) shown below, and y is a function of x

$$xy' + y - y^3 = 0.$$

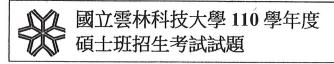
Please find the general solution of the O. D. E.

Prob. 2 (25%)

Please solve the O.D.E. with initial conditions shown below, and y is a function of t:

$$y'' + y = f(t), \quad y(0) = 1, y'(0) = 0$$

where 
$$f(t) = \begin{cases} t, & \text{for } 0 \le t < 10 \\ 1, & \text{for } 10 \le t \end{cases}$$



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## Prob. 3 (20%)

Three vectors in R<sup>4</sup> are given as below.

$$\vec{e}_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} -6 \\ 42 \\ 24 \\ 54 \end{bmatrix} \text{ and } \vec{e}_3 = \begin{bmatrix} 21 \\ -21 \\ 0 \\ -15 \end{bmatrix}$$

Are those vectors linearly independent? Why?

(Hint: If they are not linearly independent, one may find x and y, such that  $\vec{e}_3 = x\vec{e}_1 + y\vec{e}_2$ .)

## Prob. 4 (30%)

Given two straight lines and a vector A in  $\mathbb{R}^3$  space:

$$\begin{cases} x = t \\ y = -t \text{ 與} \\ z = 2t \end{cases} \qquad \begin{cases} x = 2s \\ y = s \text{ 以及 } A = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

- (i) Please find the equation of plane which contains the given two straight lines. (15%)
- (ii) Then, find the vector which is projected from vector A onto the plane obtained in (i). (15%)