



## Prob. 1 (25%)

Consider the ordinary differential equation (O.D.E.) shown below, and  $y$  is a function of  $x$

$$xy' + y - y^3 = 0.$$

Please find the general solution of the O. D. E.

## Prob. 2 (25%)

Please solve the O.D.E. with initial conditions shown below, and  $y$  is a function of  $t$ :

$$y'' + y = f(t), \quad y(0) = 1, y'(0) = 0$$

$$\text{where } f(t) = \begin{cases} t, & \text{for } 0 \leq t < 10 \\ 1, & \text{for } 10 \leq t \end{cases}$$



Prob. 3 (20%)

Three vectors in  $\mathbb{R}^4$  are given as below.

$$\bar{e}_1 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \quad \bar{e}_2 = \begin{bmatrix} -6 \\ 42 \\ 24 \\ 54 \end{bmatrix} \quad \text{and} \quad \bar{e}_3 = \begin{bmatrix} 21 \\ -21 \\ 0 \\ -15 \end{bmatrix}$$

Are those vectors linearly independent? Why?

(Hint: If they are not linearly independent, one may find  $x$  and  $y$ , such that

$$\bar{e}_3 = x\bar{e}_1 + y\bar{e}_2.)$$

Prob. 4 (30%)

Given two straight lines and a vector  $A$  in  $\mathbb{R}^3$  space :

$$\begin{cases} x = t \\ y = -t \\ z = 2t \end{cases} \quad \text{與} \quad \begin{cases} x = 2s \\ y = s \\ z = -s \end{cases} \quad \text{以及} \quad A = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

- (i) Please find the equation of plane which contains the given two straight lines. (15%)
- (ii) Then, find the vector which is projected from vector  $A$  onto the plane obtained in (i). (15%)