



1. Prove Bernoulli's inequality  $(1+x)^n > 1+nx$  for  $n = 2, 3, \dots$ , if  $x > -1, x \neq 0$ . (10 分)
2. For what values of  $x$  in the domain of definition is each of the following functions continuous? (i)  $f(x) = \frac{1+\cos x}{3+\sin x}$ . (ii)  $f(x) = \begin{cases} 10^{\frac{-1}{(x-3)^2}}, & \text{when } x \neq 3; \\ 0, & \text{when } x = 3. \end{cases}$  (10 分)
3. Show that the series  $1-1+1-1+1-1+\dots = \sum_{n=1}^{\infty} (-1)^{n-1}$  diverges. (5 分)
4. If  $F(x) = (e^{3x} - 5x)^{\frac{1}{x}}$ , find (a)  $\lim_{x \rightarrow \infty} F(x)$  and (b)  $\lim_{x \rightarrow 0} F(x)$ . (10 分)
5. Evaluate  $\int_b^a \frac{dx}{x}$ . (5 分)
6. If  $z = e^{xy^2}, x = t \cos t, y = t \sin t$ , compute  $dz/dt$  at  $t = \frac{\pi}{2}$ . (5 分)
7. Find the area of the triangle with vertices at  $P(2,3,5), Q(4,2,-1), R(3,6,4)$ . (5 分)
8. Two sides of a triangle have lengths  $a = 5$  cm and  $b = 10$  cm, and the included angle is  $\theta = \frac{\pi}{3}$ . If  $a$  is increasing at a rate of  $2$  cm/sec,  $b$  is increasing at a rate of  $1$  cm/sec, and  $\theta$  remains constant, at what rate is the third side changing? Moreover, at what rate is the area of the triangle changing? (10 分)
9. Let  $z \equiv f(x+cy) + g(x-cy)$ , where  $c \neq 0$ . Calculate  $\frac{\partial^2 z}{\partial y^2} - c^2 \frac{\partial^2 z}{\partial x^2} = ?$  (10 分)
10. The managers of a pension fund have invested \$1.5 million in Taiwan government certificates of deposit (CDs) that pay interest at the rate of 9.5% per year compounded semiannually over a period of 10 year. At the end of this period, how much will the investment be worth? What is the interest earned in the period of time? (10 分)
11. Find all relative extrema of  $x^2 y^2$  subject to the constraint  $4x^2 + y^2 = 8$ . (10 分)
12. Find the surface area generated by revolving the curve  $y = \sqrt[3]{4x}, 0 \leq y \leq 2$  about the  $y$ -axis. (10 分)



1. 有三位學生甲、乙、丙進行三個學科 A、B、C 的考試，成績如下：

學生	A	B	C
甲	92	57	77
乙	83	94	63
丙	68	73	91

若使用 F 檢定來檢定三位學生的學習能力是否相同，則 F 統計量的值為\_\_\_\_?(10 points)

2. 假設有以下 6 筆資料：

Y	8	1	20	5	50	8
X	3	1	5	3	10	7

考慮以下兩模型：

模型 A:  $Y = \alpha + \beta X + u$

模型 B:  $Y = \alpha + \beta X^2 + u$

請根據兩模型之判定係數來決定你選擇的模型是?\_\_\_\_ (10 points)

3. 假設吾人想檢定一枚銅板是否均勻，以拋擲該銅板三次之結果作為檢定方式。若三次均出現人像或三次均非為人像，則拒絕虛無假設。此檢定方式的型 1 誤差機率為\_\_\_\_。(10 points)

4. Assume that  $\mu = E(X) = 24.43$  and  $\sigma^2 = Var(X) = 2.20$ . Let  $\bar{X}$  be the sample mean of a random sample of  $n=30$ . Find  $Var(\bar{X}) =$ \_\_\_\_.(10 points)

5. Which of the following has the problem of perfect collinearity?\_\_\_\_ (10 points)

(A)  $Y = \alpha + \beta X + \gamma\left(\frac{1}{X}\right) + U$

(B)  $Y = \alpha + \beta X + \gamma(X^2) + U$

(C)  $Y = \alpha + \beta(\ln X) + \gamma(\ln X^2) + U$



6. In the past decade intensive antismoking campaigns have been sponsored by both federal and private agencies. Suppose the American Cancer Society randomly sampled 1,500 adults in 1992 and then sampled 1,750 adults in 2002 to determine whether there was evidence that the percentage of smokers had decreased. The results of the two sample surveys are shown in the table, where  $x_1$  and  $x_2$  represent the numbers of smokers in the 1992 and 2002 samples, respectively.

1992	2002
$n_1=1,500$	$n_2=1,750$
$x_1=555$	$x_2=578$

Do these data indicate the fraction of smokers decreased over this 10-year period? To answer the research question, compute the value of the appropriate test statistic.

\_\_\_\_\_ (5 points). What is the critical value of the test statistic at  $\alpha = 0.05$ ?

\_\_\_\_\_ (5 points)

7. Solve the following system:

$$\begin{bmatrix} 1 & -1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 8 \end{bmatrix}.$$

$x_1 =$  \_\_\_\_\_, (5 points) and  $x_3 =$  \_\_\_\_\_. (5 points)

8. Let  $x$  be a continuous random variable with p.d.f.  $f(x) = 3e^{-3x}$  for  $x > 0$  and  $f(x) = 0$  otherwise. Compute  $F(1) - F(0.5) =$  \_\_\_\_\_. (10 points)

9. The average number of donuts sold in a donut shop between 8 a.m. and 9 a.m. is 100 with a variance of 25. Assume we know just the two summary numbers. What is the probability that on a given day the number of donuts sold between 8 a.m. and 9 a.m. is between 90 and 110? \_\_\_\_\_ (10 points)

10. Three different machines are used to produce chocolate chip cookies by Mima's Cookie Company, which promises to have at least six chips in every cookie. Suppose machine No. 1 produces 20% of Mima's cookies, No. 2 produces 30% and No. 3 produces 50%. Also suppose that the machines represent different vintages of capital so that 1% of the cookies produced by machine No. 1 are defective, in the sense that they have less than six chips, 2% of those produced by machine No. 2 are defective, and 3% of those produced by No. 3 are defective. If one cookie is chosen at random and observed to be defective, what is the probability that it was produced by machine No. 2? \_\_\_\_\_ (10 points)



本試題共有五大計算題，每題的配分如各題的開頭所顯示。

1. (20 points) Eric receives utility from days spent traveling on vacation domestically (D) and days spent traveling in a foreign country (F) as given by the utility  $U(D, F) = DF$ . The price of a day spent traveling domestically is \$160 and in a foreign country \$200. Eric's annual budget for traveling is \$8,000.
- (a) (5 points) Find Eric's utility maximizing choice of days traveling domestically and in a foreign country. Find also his utility level from consuming that bundle.
- (b) (5 points) Suppose that the price of domestic traveling increases to \$250 per day. Calling his budget for traveling  $x$ , (suppose by now that it is unknown) find the demand for D and F under the new prices as a function of  $x$ .
- (c) (4 points) Find the income necessary to make Eric reach the same utility level as before the price change.
- (d) (6 points) Compute the quantities demanded with the new prices and the income you found in section c. Compute also the quantities demanded with the new prices and the original income. Using your answers tell us what is the total change in quantity of D due to the price increase in  $P_D$  that the consumer experiences and what part of that change is due to income or substitution effects.
2. (15 points) Molly's company produces knee warmers according to the following production function:  $q = (K-8)^{1/4} L^{1/4}$
- (a) (5 points) Assuming that the unit cost of capital ( $r$ ) and the unit wage ( $w$ ) are both equal to 1, derive Molly's demand for inputs—capital and labor, respectively—as a function of her choice of output ( $q$ ):
- (b) (2 points) Show that Molly's long run total cost function is given by
- $$C(q) = 8 + 2q^2.$$
- The demand for knee warmers is given by  $P = 40 - Q^d$ . There are no costs of entry or exit for a firm on the market for knee warmers. Any firm in this market will have access to the same technology as Molly:
- (c) (5 points) What will the price be in the long run in this market? How much will each firm produce in this market in the long run.
- (d) (3 points) How many firms will there be in this market in the long run?



3. (15 points) Suppose that Intel has a monopoly in the market for computer chips. In order to produce  $X$  computer chips, it costs Intel  $C(X) = 2X^2$ .
- (a) (2 points) Find the marginal cost of producing a computer chip for Intel.
- (b) (5 points) The demand for computer chips is  $X_D = 12 - 0.25P$  (i.e.  $P = 48 - 4X$ ). Find the level of output that maximizes Intel's profits. What price is Intel charging?
- (c) (3 points) What level of output would maximize total surplus in the computer chip market?
- (d) (5 points) If the government subsidized Intel  $s$  for every unit of computer chips produced, what quantity would Intel choose as a function of  $s$ ? Find the choice of subsidy that maximizes total surplus, i.e., induces Intel to produce the efficient quantity from part (c).

(第4題緊接在下一頁)



4. (18 points) In a far away country, the total population is 1000 people (all of them are non institutional civilian people), 564 are working and 36 are looking for a job.

Assume that firms produce goods using labor as the only factor of production. The production function is written as follows:

$$Y = 2 * N$$

where Y is output, and N is employment.

The wage setting process is described by

$$W = P^e * (Z - 200 * u)$$

where W is the nominal wage,  $P^e$  is expected price level, Z is the unemployment insurance provided by the government, and u is the unemployment rate.

Firms set their price according to

$$P = (1 + \mu) * \frac{W}{2},$$

where P is the price level, and  $\mu$  is the markup of the price over the cost. In this economy, the markup level is assumed to be 1 (i.e.  $\mu = 1$ ).

(a) (9 points) For  $P^e = P$ , what is the natural unemployment rate and the natural level of output, if  $Z=10$ ?

(b) (9 points) The government is running a fiscal surplus and they are debating whether to increase the unemployment benefits. If the government increases the unemployment benefit such that Z is increased from 10 to 13, what will be the new natural unemployment rate and the new natural level of output?



5. (32 points) Consider two open economies, Bedostan and the Republic of Deballeria. Assume that these countries only trade with each other. Variables with subscript B and variables with subscript D correspond to Bedostan and the Republic of Deballeria, respectively. The two economies are characterized by the following set of equations:

$$C_i = c_{0i} + c_{1i} * (Y_i - T_i)$$

$$I_i = \bar{I}$$

$$G_i = \bar{G}_i$$

$$T_i = t_i * Y_i$$

$$IM_i = im_{0i} + im_{1i} * Y_i$$

$$\varepsilon = 1$$

where C is consumption; Y is income; T represents taxes; I is investment; G is government spending; IM is imports; and  $\varepsilon$  is real exchange rate, the price of Deballeria goods in terms of Bedostan goods;

$i = B$  or  $D$  ( $B$  for Bedostan and  $D$  for Deballeria); and  $c_{0i}$ ,  $c_{1i}$ ,  $\bar{I}$ ,  $\bar{G}_i$ ,  $t_i$ ,  $im_{0i}$ ,  $im_{1i}$  are constant.

Let:  $c_{0B} = c_{0D} = 200$ ,  $c_{1B} = c_{1D} = 0.5$ ,  $\bar{I} = 250$ ,  $G_B = 114$ ,  $G_D = 120$ ,

$t_B = t_D = 0.4$ ,  $im_{0B} = im_{0D} = 40$ ,  $im_{1B} = 0.05$ ,  $im_{1D} = 0.3$ ,  $\varepsilon = 1$

- (8 points) Calculate the equilibrium levels of output in the two countries.
- (8 points) Calculate the trade balance for each country.
- (8 points) Suppose the government of Bedostan wants to increase government spending by 147. What will be the new equilibrium output levels in the two economies?
- (4 points) Assume that exports are exogenously given. What is the open economy multiplier in Bedostan?
- (4 points) Suppose the two economies decide to close (ie. no trade with each other). What is the multiplier in Bedostan now, assuming all the other figures remained the same?



簡答題：(100 分)

1. 請描述這次政府發放『消費券』的功能(十五分)。
2. 請定義『經濟衰退』與『經濟蕭條』(十五分)。
3. 請敘述美、日、英、台與大陸之基本放款利率為何(二十分)。
4. 金融風暴至今，大眾開始質疑分析師過去對企業的評等是否可信。請閱讀下列文字並回答問題：

「分析師無法百分之百準確預測未來股價走勢。一般而言，分析師對於市場前景、公司發展策略與訂單狀況都非常瞭解，並據以提出其對公司股價的看法，但不代表公司股價就會如其預測，畢竟這不是效率市場，分析師也無法知道所有訊息。但，很不幸的，投資人大都非常崇拜分析師，尤其是王牌分析師，更經常被投資人奉為天神，因此一旦某些時候不準，便為投資人罵到不行。」

- (1) 何為效率市場？(十五分)
  - (2) 您認為投資人看分析師報告時應注意什麼？(十分)
5. 「新興亞洲各國股價指數持續下探，和過去 5 年歷史本益比相比，不論是新興亞股，還是台、韓、印度，目前本益比都已低於過去平均值，但是，投資專家卻說：還不是進場時機」。
- 請參考下列表格資料回答問題：

	目前本益比	長期平均本益比
韓國	9.94	21.46
台灣	8.28	15.73
印度	10.85	18.78
拉丁美洲	8.40	12.20
巴西	10.10	14.59

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- (1) 本益比如何計算？代表意義？(十五分)
- (2) 若擬在韓國與台灣選擇某一市場進行投資時，此本益比資訊對您的投資決策有何影響？(十分)