



(證明題, 每題 6.25 分) - 證明題須列出完整演算及邏輯過程

- Using the following *definition of right-hand limit*, prove that  $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$   
 [definition of right-hand limit:  $\lim_{x \rightarrow a^+} f(x) = L$ : If for every number  $\varepsilon > 0$  there is a number  $\delta > 0$ , such that if  $0 < x - a < \delta$  then  $|f(x) - L| < \varepsilon$  ]
- $f$  and  $g$  are differentiable and  $g'(x) \neq 0$  on an open interval  $I$  that contains  $a$ . Suppose that  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = 0$ . Then,  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$ .  
 [提示:  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  ]

(計算題, 每題 6.25 分 - 計算題須列出完整演算過程)

- Find  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 4} - 2}{x^2}$ .
- Find the second derivative ( $f''(x)$ ) of the function  $f(x) = \sqrt{x^2 + 1}$ . Express the fraction in simplest form.
- $x^2 - xy + y^3 = 3$ , find the implicit differentiation  $\frac{dy}{dx}$
- $f(x) = 2x^3 + 3x^2 + 7x + 4$ , find  $(f^{-1})'(4)$   
 [提示: Differentiable inverse function at  $a$ :  $(f^{-1})'(a) = 1/f'(f^{-1}(a))$ ]
- Differentiate the function  $y = \frac{(e^x + e^{-x})}{(e^x - e^{-x})}$ . Express the fraction in simplest form. [提示:  $(f/g)' = (gf' - fg')/g^2$ ]
- Find the limit  $\lim_{x \rightarrow -\infty} (x^2 e^x)$  [提示: L'Hospital Rule:  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$  ]
- Determine whether the series is absolutely convergent, conditionally convergent, or divergent.  

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$
- Find the Taylor series for  $f(x) = 1/\sqrt{x}$  at  $a = 9$ .
- Evaluate the integral  $\int_{-\infty}^0 x e^x dx$ .
- Evaluate the integral  $\int \frac{\sqrt{x+4}}{x} dx$ .



13. Find the solution of the initial-value problem  $x^2 \frac{dy}{dx} + xy = 1 \quad x > 0, \quad y(1) = 2$

14. (證明題) Show that the function  $u(x, y) = e^x \sin y$  is a solution of Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

15. Find the directional derivative of the function  $f(x, y) = x^2 y^3 - 4y$  at the point  $(2, -1)$  in the direction of the vector  $\mathbf{v} = 2\mathbf{i} + 5\mathbf{j}$ .

16. Evaluate the double integral  $\iint_R (x - 3y^2) dA$ , where  $R = \{(x, y) \mid 0 \leq x \leq 2, 1 \leq y \leq 2\}$ .


**壹、選擇題 (共七題，合計50分)**
**選擇題說明：**

1. 選擇題共7題。4題複選題(題號：A-1至A-4)，3題單選題(題號：A-5至A-7)。
2. 複選題每題8分(共32分)，單選題每題6分(共18分)。第壹部分選擇題合計50分。
3. 單選題請以最接近的答案選答，複選題須完全答對該題才計分。

A-1 (複選題) 對於各種預測方法及特性，下列敘述何者正確?

- (A) 採線性趨勢方程式(Linear Trend Equation)所得的預測值有可能大於過去所有的實際值
- (B) 採三期為基礎的移動平均法，其預測值必定小於或等於最近三期實際值之最大者
- (C) 採三期為基礎的加權移動平均法，其預測值必定大於或等於最近三期實際值之最小者
- (D) 採指數平滑法做預測時，若採用較大的 $\alpha$ 值將會造成預測值較小的波動
- (E) 以時間序列資料為基礎的各種預測方法，其預測值對於未來事件的影響無法納入考慮

A-2 (複選題) 有關生產線平衡的方法及現象，下列敘述何者正確?

- (A) 各種生產線平衡技術所安排的生產線，均可以達到期望的產出量亦可遵守先行作業要求
- (B) 各工作站作業時間之最短者，其作業時間即為生產線的實際週期時間(Actual Cycle Time)
- (C) 各工作站作業時間之最長者，該工作站即可稱為生產線的瓶頸工作站
- (D) 各種生產線平衡的方法，均可以達到理論上的最佳解
- (E) 若要增加已排定之生產線的產量，則應選擇瓶頸工作站、並減少該工作站的作業時間

A-3 (複選題) 某產品期初庫存 100，生產批量為 90。依照下表的四期資料，以 Master Scheduling 程序進行分析，並依照 Master Scheduling 的結果答覆問題。下列敘述何者正確?

	Period			
	1	2	3	4
Forecast	60	60	50	50
Customer Order	65	50	20	10

- (A) 第 1 期至第 4 期的 ATP 總和為 135
- (B) 第 1 期至第 4 期的 MPS 依序分別為：0, 90, 0, 90
- (C) 第 4 期末的預計存貨(Projected On-hand)量為 50
- (D) 第 1 期至第 4 期中，只有第 3 期需生產一個批量
- (E) 表中 Customer Order 及 MPS 均不改變之下，若另收到一張訂單(於第 3 期交貨 50)，則該訂單可以如期交貨



A-4 (複選題) 有關物料需求計劃(Material Requirement Planning, MRP)的敘述，下列何者正確?

- (A) Lead Time 資料是 MRP 的產出(Output)資料之一
- (B) MRP Processing 之前，必須先確定：採用 Lot-for-Lot Ordering 或採用 Lot-Size Ordering
- (C) MRP Processing 整體的推導順序為：由完成品的 MPS 開始，推導至各零組件的製造時程，最後推導至原物料的採購時程
- (D) MRP Processing 每階層物料的數量推導時，是先由淨需求(Net Requirement)再逐步推算出毛需求(Gross Requirement)
- (E) MRP Processing 每階層物料的時程推導時，是先由計劃訂單接收量(Planned-Order Receipt)再推導出計劃訂單發出量(Planned-Order Release)

A-5 (單選題) 大量客製化(Mass Customization)的理念，可以透過下列何種方式完成?

- (A) 品質機能展開(Quality Function Deployment)及重新製造(Remanufacturing)
- (B) 穩健設計(Robust Design)及可靠度(Reliability)分析
- (C) Kano 模型分析及價值分析(Value Analysis)
- (D) 模組設計(Modular Design)及延遲差異化(Delayed Differentiation)
- (E) 生命週期評估(Cradle-to-grave Assessment)及反向工程(Reverse Engineering)

A-6 (單選題) 某產品加工過程之中需經過兩次非連續的鍛造加工，每次鍛造所需時間均為每件 S 小時，第一次鍛造不良率為 X%、第二次鍛造不良率為 Y%。每台鍛造機的設計產能為每年 P 小時、有效產能為每年 Q 小時、去年鍛造機的實際產出工時為 R 小時。明年該產品的產量預估為 Z 件/年，試預估明年鍛造機的需求台數為何?

- (A)  $[ZS/(1-Y\%) + ZS/(1-Y\%)(1-X\%)] / Q$
- (B)  $[ZS/(1-Y\%) + ZS/(1-X\%)] / P$
- (C)  $[ZS/(1-Y\%) + ZS/(1-X\%)] / Q$
- (D)  $[2ZS/(1-Y\%-X\%)] / R$
- (E)  $[ZS/(1-X\%) + ZS/(1-Y\%)(1-X\%)] / R$

A-7 (單選題) 下列有關定量訂購系統或定期訂購系統的敘述，何者正確?

- (A) 定量系統的再訂購點(Reorder Point)即為最佳訂購量的二分之一
- (B) 定期系統的運作方式為：當存貨量低於或等於再訂購點時，則必須立即發出一個訂單
- (C) 定量系統的再訂購點 = 訂購前置期(Lead Time)內的預期需求量 + 平均庫存量
- (D) 定期系統的服務水準(Service Level)愈高，則訂購量也愈多
- (E) 若僅從定量系統及定期系統兩者擇一，則複倉制(Two-Bin System)的運作方式與定量系統較為近似



## 貳、計算題 (共三題，合計 50 分)

1. (15 分) 某公司對於所生產產品 A 之部分零件以經濟生產批量法 (Economic Production Quantity, EPQ) 作為該零件存貨控管之模式。經過生管工程師詳細估算後所需之資料如下：

生產率：1200/天                      年需求量：200000 個    生產整備成本：200/次  
 單位存貨成本：1 個/年            安全存量：200 個        每年天數：250 天

- (a) 最大存貨量為若干。(8 分)  
 (b) 僅消耗而不生產之時間為何。(7 分)

2. (20 分) 某汽車鈹金工廠有 5 部汽車需要維修，小張是鈹金工廠之領班，他必須全程在廠督工。假設每部汽車需要經過一、鈹金加工，二、粉底烤漆，每部汽車在各個部門預估加工時間如下表所示。若小張欲以總完工時間 (makespan) 最短來安排 5 輛汽車維修之順序，試回答下列問題。

汽車	鈹金加工	粉底烤漆
A	1.7	2.0
B	2.5	1.5
C	1.6	2.5
D	2.0	1.1
E	1.0	1.4

- (a) 小張最少需要在工廠督工多久。(8 分)  
 (b) 粉底烤漆工之工作時間多久 (從開始工作至結束工作含閒時)。(5 分)  
 (c) 如果在第一部汽車鈹金完工後，小張發現 C 部汽車工時預估有誤，而加以重新更正，且時間為

汽車	鈹金加工	粉底烤漆
C	2.5	1.3

則 C 車之完工時間與原先預估之完工時間差距若干。(7 分)

3. (15 分) 一物料於未來 9 週之需求及各項相關資料如下，請用單位最低成本法 (Least Unit Cost, LUC) 決定第一次訂購時間、與訂購數量。

週別	1	2	3	4	5	6	7	8	9
需求量	50	30	10	40	35	25	15	30	30

- 該物料之單價為 \$50，倘若一次購買量超過 60 個則超出部份單價為 \$45。
- 訂購一次之成本為 \$2000，
- 單位存貨成本為 \$5/個/期
- 期初存貨為 60 個，且前置時間為 1 週



You need to define and explain your notations clearly. Points are granted only if detail computations are presented.

1. Consider the following problem:

$$\begin{aligned}
 \max \quad & z = x_1 + x_2 - 2x_3 \\
 \text{s.t.} \quad & x_1 + x_2 + 2x_3 \leq 100 \\
 & x_1 - 2x_2 \geq 10 \\
 & \frac{x_3}{x_1 + x_2} \geq \frac{1}{2} \\
 & x_1, x_2, x_3 \geq 0.
 \end{aligned}$$

- (a) Solve the problem by Simplex algorithm. (15%)  
 (b) What are the reduced costs and shadow prices of the problem at optimality. (10%)
2. YunTech Transport is to deliver products from three factories to three sales centers. Information about the transportation is given below. A per unit shortage cost \$2 incurs for any unsatisfied demand, and a per unit holding cost \$1 incurs for any inventory.

Table 1: Information about the transportation

(a) Amounts of supply and demand				(b) Unit delivery cost				
	Supply		Demand	Center				
Factory 1	100	Center 1	130					
Factory 2	150	Center 2	100					
Factory 3	120	Center 3	80					
		Center 4	120					
				1	2	3	4	
				Factory 1	3	2	5	4
				Factory 2	9	10	8	20
				Factory 3	5	6	4	10

- (a) Formulate YunTech Transport's problem as a linear programming model. (10%)  
 (b) Solve the problem (clearly show the basic variables and the objective value at optimality) (15%)
3. Consider a car repair shop with 2 repairmen and 2 waiting spaces for cars (not including spaces for repairing cars). The arriving failed car process is a Poisson process with rate 2 per hour. The car repair time is exponential with mean time of



20 minutes. The arriving car that can not find the a space to wait leave the shop immediately.

- (a) Find the expected number of waiting cars.(10%)
  - (b) Find the proportion of cars that leave repair shop without being repaired.(10%)
  - (c) If the average revenue generated by repairing a car is \$100, and cost of a repairman while repairing car is \$50 per hour, find the average net profit per hour of the car repair shop in steady state. (10%)
4. Assume that the yield of planting apple in a year is categorized into three types: good (G), average (A) and bad (B). When the yield is good, the revenue generated from apple is \$8000. When the yield is average, the revenue generated from apple is \$12000. When the yield is bad, the revenue generated from apple is \$6000. Assume that yield of apple this year depends on yield of apple last year. Let  $X_n$  be the yield of apple in year n. The changes of yield in each year have the following transition probabilities :  $P(X_{n+1} = G|X_n = A) = P_{AG} = 0.2$ , following with similar notation definition,  $P_{AA} = 0.3$ ,  $P_{AB} = 0.5$ ,  $P_{GG} = 0.5$ ,  $P_{GA} = 0.4$ ,  $P_{GB} = 0.1$ ,  $P_{BG} = 0.2$ ,  $P_{BA} = 0.3$ ,  $P_{BB} = 0.5$ . Find the average revenue per year of apple in steady state. (20%)



## (一) 選擇題：單選題，每題 5 分。

1. A business graduate has applied for a job with two insurance firms : **A** and **B**. The graduate feels that she has a 65% chance of receiving an offer from firm **A** and a 50% chance of receiving an offer from firm **B**. If she receives an offer from firm **B**, she believes that she has a 90% chance of receiving an offer from firm **A**. What is the probability that both firms will make her an offer?  
 (A) 0.65      (B) 0.5      (C) 0.45      (D) 0.9      (E) 0.25
  
2. Men have a reputation for not wanting to ask for directions. A study conducted for Yuntech Company indicated that 42% of men and 61% of women would stop and ask for directions. The government's 2017 population estimate was that for adults 48.2% were men and 51.8% were women. Given that a driver stops to ask for directions, determine the probability that the driver was a man.  
 (A) 0.518      (B) 0.420      (C) 0.316      (D) 0.390      (E) 0.7778
  
3. A shipment of 20 electronic components was sent to Yuntech Company. Four of them were defective. One of the company technicians selected 5 of the electronic components to put in his inventory, and went on three service call. Determine the probability that 3 of the 5 electronic components are not defective.  
 (A) 0.05      (B) 0.2167      (C) 0.3163      (D) 0.4696      (E) 0.968
  
4. It is assumed that the time failures for an electronic component are exponentially distributed with a mean of 50 hours between consecutive failures. Based on this information, what is the probability that a randomly selected part will fail in less than 10 hours?  
 (A) 0.82      (B) 0.20      (C) 0.33      (D) 0.27      (E) 0.18

## (二) 計算題。

5. (10%) A year-old study found that the service time for all drive-thru customers at a coffee Shop is uniformly distributed between 4 and 10 minutes. A random sample of 49 customers is taken and the service time for each is recorded.  
 (A) Calculate the mean and standard deviation of service times for all drive-thru customers at this Coffee Shop.





- (B) What is the probability that a sample of 49 customers would have a sample mean of 6.5 minutes or more?
6. (10%) The manager of a restaurant knows from experience that 60% of the people who make reservations for the evening show up for dinner. The manager decides one evening to overbook and accept 20 reservations when only 15 tables are available. What is the probability that more than 15 parties will show up?
7. (10%) Monthly sales of a certain product, recorded in nearest thousand, are believed to follow the probability distribution given in below table.

Number of items	Probability
5000	0.2
6000	0.3
7000	0.2
8000	0.2
9000	0.1

- (A) What is the expected number of items sold monthly?
- (B) Suppose that the company has a fixed monthly production cost of \$8000 and that each item bring \$2. Find the expected monthly profit from product sales.
8. (10%) When  $X_1, \dots, X_n$  are independent Poisson random variables, each with parameter  $\lambda$ , and when  $n$  is relatively large, the sample mean  $\bar{X}$  is approximately normal with mean  $\lambda$  and variance  $\lambda/n$ .
- (A) What is the distribution of the statistic  $\frac{\bar{X}-\lambda}{\sqrt{\lambda/n}}$
- (B) Use the results of (a) to find a  $100(1 - \alpha)\%$  confidence interval for  $\lambda$ .
9. (10%) A pseudorandom number generator is designed so that the integers 0 through 9 have equal probability of occurrence. The first 10,000 numbers are:

0	1	2	3	4	5	6	7	8	9
967	1008	975	1022	1003	989	1001	981	1043	1011

Does this generator seem to working properly?



10. (20%) Two methods for producing gasoline from crude oil are being investigated. The yields of both processes are assumed to be normally distributed. The following yield data have been obtained from the pilot plant.

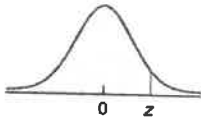
Process	Yields (%)					
1	24.2	26.6	25.7	24.8	25.9	26.5
2	21.0	22.1	21.8	20.9	22.4	22.0

- (A) Is there reason to believe that process 1 has a greater mean yield? Use  $\alpha = 0.05$ . Assume both variances are equal.
- (B) Assuming that in order to adopt process 1 it must produce a mean yield that is at least 5 percent greater than that of process 2, what are your recommendations?
- (C) Find the power of the test in part (a) if the mean yield of process 1 is 5 percent greater than that of process 2.
- (D) What sample size is required for the test in part (a) to ensure that the null hypothesis will be rejected with probability 0.90 if the mean yield of process 1 exceeds the mean yield of process 2 by 5 percent?
11. (10%) The cell phone industry is involved in a fierce battle for customers, with each company devising its own complex pricing plan to lure customers. A consumer watchdog group decided to compare the average costs for four cellular phone companies using three different usage levels as blocks. The monthly costs computed by the cell phone companies for peak-time callers at low(20 minutes per month), middle(150 minutes per month), and high(1000 minutes per month) usage levels are given in the following table. Is there a sufficient evidence to indicate a difference in the average monthly costs for the four companies? Use  $\alpha = 0.05$ .

Usage Level	Company			
	A	B	C	D
Low	27	24	31	23
Middle	68	76	65	67
High	308	326	312	300

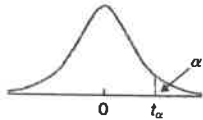


TABLE II (cont.)  
Areas under the  
standard normal curve



z	Second decimal place in z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000 <sup>†</sup>									

<sup>†</sup> For  $z \geq 3.90$ , the areas are 1.0000 to four decimal places.


 TABLE IV  
 Values of  $t_\alpha$ 


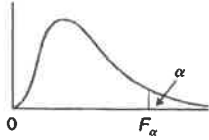
df	$t_{0.10}$	$t_{0.05}$	$t_{0.025}$	$t_{0.01}$	$t_{0.005}$	df
1	3.078	6.314	12.706	31.821	63.657	1
2	1.886	2.920	4.303	6.965	9.925	2
3	1.638	2.353	3.182	4.541	5.841	3
4	1.533	2.132	2.776	3.747	4.604	4
5	1.476	2.015	2.571	3.365	4.032	5
6	1.440	1.943	2.447	3.143	3.707	6
7	1.415	1.895	2.365	2.998	3.499	7
8	1.397	1.860	2.306	2.896	3.355	8
9	1.383	1.833	2.262	2.821	3.250	9
10	1.372	1.812	2.228	2.764	3.169	10
11	1.363	1.796	2.201	2.718	3.106	11
12	1.356	1.782	2.179	2.681	3.055	12
13	1.350	1.771	2.160	2.650	3.012	13
14	1.345	1.761	2.145	2.624	2.977	14
15	1.341	1.753	2.131	2.602	2.947	15
16	1.337	1.746	2.120	2.583	2.921	16
17	1.333	1.740	2.110	2.567	2.898	17
18	1.330	1.734	2.101	2.552	2.878	18
19	1.328	1.729	2.093	2.539	2.861	19
20	1.325	1.725	2.086	2.528	2.845	20
21	1.323	1.721	2.080	2.518	2.831	21
22	1.321	1.717	2.074	2.508	2.819	22
23	1.319	1.714	2.069	2.500	2.807	23
24	1.318	1.711	2.064	2.492	2.797	24
25	1.316	1.708	2.060	2.485	2.787	25
26	1.315	1.706	2.056	2.479	2.779	26
27	1.314	1.703	2.052	2.473	2.771	27
28	1.313	1.701	2.048	2.467	2.763	28
29	1.311	1.699	2.045	2.462	2.756	29
30	1.310	1.697	2.042	2.457	2.750	30
31	1.309	1.696	2.040	2.453	2.744	31
32	1.309	1.694	2.037	2.449	2.738	32
33	1.308	1.692	2.035	2.445	2.733	33
34	1.307	1.691	2.032	2.441	2.728	34
35	1.306	1.690	2.030	2.438	2.724	35
36	1.306	1.688	2.028	2.434	2.719	36
37	1.305	1.687	2.026	2.431	2.715	37
38	1.304	1.686	2.024	2.429	2.712	38
39	1.304	1.685	2.023	2.426	2.708	39
40	1.303	1.684	2.021	2.423	2.704	40
41	1.303	1.683	2.020	2.421	2.701	41
42	1.302	1.682	2.018	2.418	2.698	42
43	1.302	1.681	2.017	2.416	2.695	43
44	1.301	1.680	2.015	2.414	2.692	44
45	1.301	1.679	2.014	2.412	2.690	45
46	1.300	1.679	2.013	2.410	2.687	46
47	1.300	1.678	2.012	2.408	2.685	47
48	1.299	1.677	2.011	2.407	2.682	48
49	1.299	1.677	2.010	2.405	2.680	49


 TABLE VII (cont.)  
 Values of  $\chi^2_{\alpha}$ 

$\chi^2_{0.10}$	$\chi^2_{0.05}$	$\chi^2_{0.025}$	$\chi^2_{0.01}$	$\chi^2_{0.005}$	df
2.706	3.841	5.024	6.635	7.879	1
4.605	5.991	7.378	9.210	10.597	2
6.251	7.815	9.348	11.345	12.838	3
7.779	9.488	11.143	13.277	14.860	4
9.236	11.070	12.833	15.086	16.750	5
10.645	12.592	14.449	16.812	18.548	6
12.017	14.067	16.013	18.475	20.278	7
13.362	15.507	17.535	20.090	21.955	8
14.684	16.919	19.023	21.666	23.589	9
15.987	18.307	20.483	23.209	25.188	10
17.275	19.675	21.920	24.725	26.757	11
18.549	21.026	23.337	26.217	28.300	12
19.812	22.362	24.736	27.688	29.819	13
21.064	23.685	26.119	29.141	31.319	14
22.307	24.996	27.488	30.578	32.801	15
23.542	26.296	28.845	32.000	34.267	16
24.769	27.587	30.191	33.409	35.718	17
25.989	28.869	31.526	34.805	37.156	18
27.204	30.143	32.852	36.191	38.582	19
28.412	31.410	34.170	37.566	39.997	20
29.615	32.671	35.479	38.932	41.401	21
30.813	33.924	36.781	40.290	42.796	22
32.007	35.172	38.076	41.638	44.181	23
33.196	36.415	39.364	42.980	45.559	24
34.382	37.653	40.647	44.314	46.928	25
35.563	38.885	41.923	45.642	48.290	26
36.741	40.113	43.195	46.963	49.645	27
37.916	41.337	44.461	48.278	50.994	28
39.087	42.557	45.722	49.588	52.336	29
40.256	43.773	46.979	50.892	53.672	30
51.805	55.759	59.342	63.691	66.767	40
63.167	67.505	71.420	76.154	79.490	50
74.397	79.082	83.298	88.381	91.955	60
85.527	90.531	95.023	100.424	104.213	70
96.578	101.879	106.628	112.328	116.320	80
107.565	113.145	118.135	124.115	128.296	90
118.499	124.343	129.563	135.811	140.177	100



TABLE VIII  
Values of  $F_\alpha$



dfd	$\alpha$	dfn								
		1	2	3	4	5	6	7	8	9
1	0.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	0.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
	0.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	0.01	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	0.005	16211	20000	21615	22500	23056	23437	23715	23925	24091
2	0.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	0.01	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	0.005	198.50	199.00	199.17	199.25	199.30	199.33	199.36	199.37	199.39
3	0.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	0.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	0.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88
4	0.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	0.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	0.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14
5	0.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	0.05	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	0.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	0.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	0.005	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77
6	0.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	0.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	0.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	0.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	0.005	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39
7	0.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	0.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	0.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	0.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	0.005	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51
8	0.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	0.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	0.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	0.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	0.005	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34