國立雲林科技大學
97學年度碩士班入學招生考試試題

系所：通訊所
科目：機率學

1. (20%) 根據某民間機構統計，大約 54%的台灣男性成年人相信台灣會被接受加入聯合國而，只有大約 33%的台灣女性成年人相信會被接納。而根據台灣地區人口調查，48%的台灣成年人是男性。試問
   (i) (10%) 多少比例的台灣成年人相信台灣會被接受加入聯合國？
   (ii)(10%) 在相信台灣會被接受加入聯合國的成年人裡，女性成年人佔多少比例？

2. (30%)非官方版的小樂透彩券以如下規則發行：購買一張彩券可以從 1～42 個號碼裡由電腦隨機挑六個不同號碼。兌中開獎時的三個或更多號碼稱爲中獎者。【註：若你無簡單掌上型計算機又嫌計算麻煩，你可以符號代替數值但須說明運算方式】
   (i) (10%)令隨機變數 X 是一張彩券中得號碼的個數，試求 X 之機率質量函數(Probability mass function)爲何？
   (ii)(5%)假定你去買一張彩券，中獎機率有多少？
   (iii)(5%)六個號碼全中稱爲頭獎，一張彩券中頭獎機率有多大？
   (iv)(5%)假定你每週都買一張，試問你在一年(52 週)裡會得獎至少一次的機率是多少？
   (v)(5%)假定你每週都買一張，試問你剛好花一年的時間才得獎的機率是多少？

3. (20%)一隨機變數 X 的機率密度函數(probability density function)爲均勻分佈(uniformly distributed)於區間 $[a,b]$ 的函數，且 $a \neq b$，
   (i)(5%)求隨機變數 X 的機率分佈函數(probability distribution function)
   (ii)(5%)求隨機變數 X 的變異數(variance)
   (iii)(10%)求一新的隨機變數 $Y = 5X + 3$ 之變異數

4. (20%)兩隨機變數 X, Y 的平均值分別爲 $E[X] = 1, E[Y] = 3$，變異數分別爲 $\sigma_X^2 = 16$，$\sigma_Y^2 = 1$，且其相關係數爲 $\rho_{XY} = 0.3$. 定義兩個新的隨機變數 $Z = 2X + Y$ 及 $W = X - 2Y$，則
   (i)(10%)求隨機變數 W 的變異數
   (ii)(10%)求隨機變數 ZW 的平均值 $E[ZW]$

5.(10%)假設 $P(A^C) = 0.2$，$P(B) = 0.6$，且 $P(A \cap B^C) = 0.3$，求 $P(B \mid A \cup B^C)$ 的值。

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系所：通訊所
科目：通信系統

1. (15%) In a DSB-SC modulation system, the carrier is $c(t) = A_c \cos(2\pi f_c t)$, the message signal is $m(t) = \text{sinc}(t) + \text{sinc}^2(t)$.
   (a) (10%) Find the Fourier transformation of the modulated signal $s(t)$.
   (b) (5%) Determine the bandwidth of the transmitted signal.

2. (20%) The IF frequency in an AM radio is $f_{IF} = 455\,\text{kHz}$. Assume the desired signal has a carrier frequency of 600 kHz.
   (a) (8%) Find the LO frequency and the image frequency of the desired signal
   (b) (12%) Draw a block diagram of a superheterodyne receiver and explain how it can remove the image signal.

3. (20%)A sinusoidal signal $m(t) = 2\cos(2\pi 10^4 t)$ is frequency modulated with carrier frequency f_c = 100 MHz. Assume the frequency sensitivity of the modulator is $k_f = 30\,\text{kHz/V}$.
   (a) (8%) Use Carson's rule to find the transmission bandwidth of the FM signal.
   (b) (6%) How will the transmission bandwidth change if the carrier frequency is increased?
   (c) (6%) If the message signal is replaced by $m(t) = 1 + 2\cos(2\pi 10^4 t)$, find the transmission bandwidth.

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4. (20%) Consider the three functions $\phi_1(t)$, $\phi_2(t)$ and $\phi_3(t)$ shown in Figure 1.

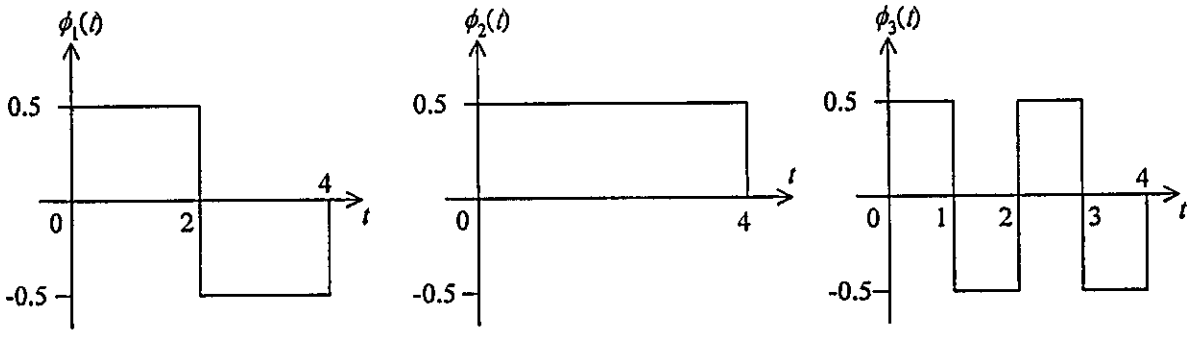

Figure 1

(a) (8%) Determine whether these three functions are orthogonal to each other over the interval [0, 4].

(b) (4%) Determine whether these three functions can form a set of orthonormal basis functions?

(c) (8%) Assume a function $x(t)$ is defined as below. Express $x(t)$ as the linear combination of $\phi_1(t)$, $\phi_2(t)$ and $\phi_3(t)$.

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 2, & 1 \le t \le 2 \\ 0, & 2 \le t \le 3 \\ 1, & 3 \le t \le 4 \end{cases}$$

5. (25%) Consider a discrete memoryless source with source alphabet $S = \{ s_1, s_2, s_3, s_4, s_5, s_6, \}$ with respective probabilities { 0.3, 0.2, 0.2, 0.1, 0.1, 0.1 }.

(a) (8%) Calculate the entropy of the source.

(b) (4%) Calculate the entropy of the second-order extension of the source.

(c) (7%) Construct the Huffman code for this source.

(d) (6%) Evaluate the average codeword length and the efficiency of the Huffman code.

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系所：通訊所、電機系
科目：線性代數

1. (10%) Let $\mathbf{u}$ be a vector in $\mathbf{R}^2$ whose projection onto the $x-axis$ is u_x as shown in Figure 1. Determine the entries of the vector $\mathbf{u}$.

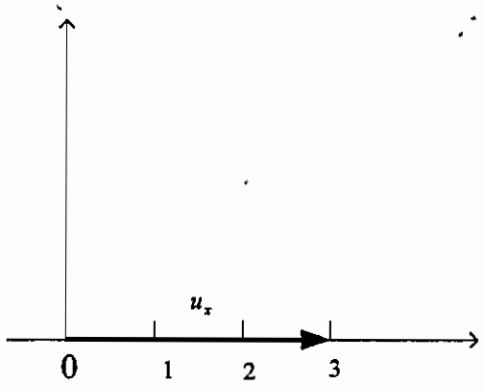

Figure 1.

2. (10%) Let

$$\mathbf{A} = \begin{bmatrix} 6 & 2 & 8 \\ 9 & 5 & 11 \\ 3 & 1 & 6 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 2 & 0 & 0 \\ t & s & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} r & 1 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & p \end{bmatrix}$$

Find scalars r, s, t and p so that $\mathbf{LU} = \mathbf{A}$.

3. (15%) Determine whether each of the following statements is *True* or *False*, and explain.

(a) $\det(A+B) = \det(A) + \det(B)$

(b) $\det\left(A^{-1}B\right) = \frac{\det(B)}{\det(A)}$

(c) If $\det(A) = 0$, then A has at least two equal rows.

(d) If A has a column of all zeros, then $\det(A) = 0$.

(e) A is singular if and only if $\det(A) = 0$.

4. (15%) Let

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 8 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Show that $span(S) = \mathbf{R}^3$ and find a basis for $\mathbf{R}^3$ consisting of vectors from S.

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系所：通訊所、電機系
科目：線性代數

5. (15%) Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation for which we know that

$$L\left(\begin{bmatrix}1\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix},\quad L\left(\begin{bmatrix}0\\1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\\0\end{bmatrix}, L\left(\begin{bmatrix}1\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

(a) Find $L\left(\begin{bmatrix}4\\1\\0\end{bmatrix}\right)$ =?

(b) Find $L\left(\begin{bmatrix}0\\0\\0\end{bmatrix}\right)$ =?

6. (10%) Consider vector space $\mathbf{R}^2$.

(a) For what values of m and b will the set of all vectors of the form $\begin{bmatrix}x\\mx+b\end{bmatrix}$ be a *subspace* of $\mathbf{R}^2$?

(b) For what value of r will the set of all vectors of the form $\begin{bmatrix}x\\rx^2\end{bmatrix}$ be a *subspace* of $\mathbf{R}^2$?

7. (10%) Let L: $\mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by

$$L\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = \begin{bmatrix}x_2 - x_1\\2x_1 + x_2\end{bmatrix},$$

and let

$$S = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$$
$$T = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}\right\}$$

be two bases for $\mathbf{R}^2$. Find the matrix representation $[L]_T^S$ of L with respect to T and S.

8. (15%) Let $A = \begin{bmatrix}3 & 1\\0 & 2\end{bmatrix}$.

(a) Find a nonsingular matrix P such that $P^{-1}AP$ is diagonal.

(b) Derive a formula for A^k, where k is any positive integer.