國立雲林科技大學
95 學年度碩士班入學招生考試試題

系所：通訊所
科目：機率學

1. (20%) Toss a fair die 4 times. Let X_i be the number that appears at the i-th toss.

(a) (10%) Calculate $P\{ X_1 < \min(X_2, X_3, X_4) \}$
(b) (10%) Calculate $P\{ X_1 < X_2 < X_3 < X_4 \}$

2. (20%) Two numbers X and Y are randomly selected from the interval [0, 1]. Define two events: $A = \{ X^2 + Y^2 > 1 \}$, and $B = \{ X > Y \}$

(a) (8%) Find $P(A)$
(b) (7%) Find $P(A \mid B)$
(c) (5%) Determine whether events A and B are independent.

3. (20%) Consider a set of n independent and identically distributed (i. i. d.) random variables $X_1, X_2, \ldots, X_n$, each uniformly distributed over [0, 1]. Let $Y = \max\{ X_1, X_2, \ldots, X_n \}$.

(a) (14%) Find the probability density function (pdf) of Y.
(b) (6%) Calculate the expected value of Y.

4. (20%) Let X, Y, and Z be independent zero-mean, normally distributed random variables. Assume their standard deviations are $\sigma_X = 1$, $\sigma_Y = 2$, and $\sigma_Z = 3$, respectively.

(a) (10%) Calculate $P(X + Y + Z < 6)$
(b) (10%) Calculate $P(3X - 2 < 2Y + Z)$
{You may express the answers in terms of the distribution function of X, $F_X(x)$.}

5. (20%) Assume the joint density function of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-x}e^{-y}, & 0 \le y < x < \infty \\ 0, & elsewhere \end{cases}.$$

(a) (10%) Find the marginal density function, $f_Y(y)$, of Y.
(b) (10%) Compute $E[Y]$ and Var[Y].

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系所：通訊所
科目：通信系統

20%第一題

10% (A) 請分別畫出 QPSK 傳送器及同調 QPSK 接收器的方塊圖。

10% (B) 請畫出 QPSK 及 Offset QPSK 之星座圖並比較兩種技術的差異性。

20%第二題

A random signal is defined as follows.

$$V(t) = A\cos(2\pi f_c t + \Theta)$$

where A and f_c are known constants and Θ is uniformly distributed between 0 and 2π.

10% (A) Calculate E[V(t)] and autocorrelation function $R_V(t_1, t_2)$

10% (B) Is V(t) wide sense stationary(WSS)? What's its power spectral density?

30%第三題

For the AWGN channel with a white noise having power spectral density of N_0, consider the peak-signal-to-rms noise ratio at the output of a matched filter for the two pulses:

$$g_1(t) = a\prod\left(\frac{t-t_0}{T}\right)$$

and

$$g_2(t) = b\cos\left[\frac{2\pi(t-t_0)}{T}\right]\prod\left(\frac{t-t_0}{T}\right)$$

where

$$\prod(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & otherwise \end{cases}$$

15%(A) Relate a and b such that both pulses provide the same signal-to-noise ratio at the matched filter output.

15%(B) For a white noise with power spectral density of N_0, and the relation of a and b in (A), please find the average error probability in terms of Q-function:

$$Q(x) = \int_x^{\infty} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$$

20%第四題

考慮一個鎖相迴路(PLL)包含一個乘法器，迴路濾波器及壓控震盪器(VCO)。令乘法器的輸入是 PSK 信號定義如下

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

其中 k_p 是相位靈敏度，$m(t)$ 是資料信號當符號值爲 1 時其爲+1，符號值爲 0 時其爲-1。壓控震盪器(VCO)的輸出是

$$r(t) = A_c \sin[2\pi f_c t + \theta(t)]$$

10% (A) 計算迴路濾波器的輸出，假設這濾波器只移動載波 $2f_c$ 調變部份。

5% (B) 當 k_p 值爲多少，鎖相迴路將沒有載波的成份可以追蹤？

5% (C) 當迴路相位鎖住時，該迴路濾波器的輸出與資料信號 $m(t)$是什麼關係？

10%第五題

From the experience of Fourier series expansion (or Fourier Transform), we learned that we can view the non-random time functions as the weighted sum of the integral of the sinusoidal functions. The rate at which a non-random time function varies is related to the weighting functions (spectrum, so called) of the Fourier series or transform. How about the stationary random signals?(Explain how we can measure its average rate of change of the ensemble of those sample functions?)