



1. (15%) Show that the matrices  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

satisfy the commutation relation (a) (5%)  $[A, B] = C$

(b): (5%)  $[A, C] = 0$

(c): (5%)  $[B, C] = 0$

2. (15%) A function  $f(x) = e^{-a|x|}$  ( $a > 0$ ), find (a) (10%) Fourier Integral of  $f(x)$

and (b) (5%) calculate  $\int_b^{\infty} \frac{\cos(2x)}{x^2 + 4} dx$

3. (10%) Please solve  $x^2 y'' - 13xy' + 49y = 0$

4. (10%) Please evaluate  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

5. (12%) If  $z_1 = i$ ,  $z_2 = 1 - \sqrt{3}i$ , please find (1)  $\arg\left(\frac{z_1}{z_2}\right)$  and (2)  $\arg(z_1 z_2)$

In problems (a), express  $e^z$  in the form  $a + ib$

6. (12%) (a)  $z = -\pi + \frac{3\pi}{2}i$

In problems (b), express  $\ln z$  in the form  $a + ib$

(b)  $z = -2 + 2i$

7. (13%) Please solve  $y^2 \frac{dx}{dy} + 2yx = x^4$

8. (13%) Please find the general solution of the P.D.E.  $\frac{\partial^2 Z}{\partial x^2} - \frac{\partial^2 Z}{\partial y^2} - \frac{\partial Z}{\partial x} + \frac{\partial Z}{\partial y} = 0$



說明：本試卷共六大題，總分共計 100 分。

1. The current  $I$  (A) flows from a point charge  $Q(t)$  at the origin to infinity along a semi-infinitely long, straight wire occupying the positive  $z$ -axis, and find

$$\oint_C \vec{H} \cdot d\vec{l}$$

where  $C$  is a circular path of radius  $a$  lying in the  $xy$ -plane and centered at

the point charge, as shown in Fig.1. (15%)

2. In Fig. 2, the region  $x < 0$  is a perfect conductor, the region  $0 < x < d$  is a perfect dielectric of  $\epsilon = 4\epsilon_0$  and  $\mu = \mu_0$ , and the region  $x > d$  is free space. The electric and magnetic fields in the region  $0 < x < d$  are given at a particular instant of time by

$$\mathbf{E} = E_1 \cos \pi x \sin 2\pi z \mathbf{a}_x + E_2 \sin \pi x \cos 2\pi z \mathbf{a}_z$$

$$\mathbf{H} = H_1 \cos \pi x \sin 2\pi z \mathbf{a}_y$$

Find (a)  $\rho_s$  and  $\mathbf{J}_s$  on the surface  $x = 0$  (7%) and (b)  $\mathbf{E}$  and  $\mathbf{H}$  for  $x = d+$  (8%), that is immediately adjacent to the  $x = d$  - plane and on the free-space side, at that instant of time.

3. A current  $I$  (A) flows with uniform volume density  $\mathbf{J} = J_0 \mathbf{a}_z$  (A/m<sup>2</sup>) along an infinitely long, solid cylindrical conductor of radius  $a$  and returns with uniform surface density in the opposite direction along the surface of an infinitely long, perfectly conducting cylinder of radius  $b$  ( $> a$ ) and coaxial with the inner conductor. Try to find the internal inductance per unit length of the inner conductor. (See Fig.3) (20%)

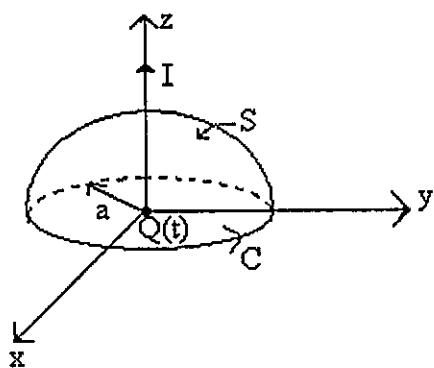


Fig.1.

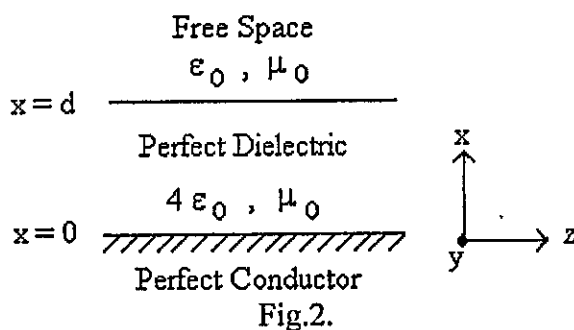


Fig.2.

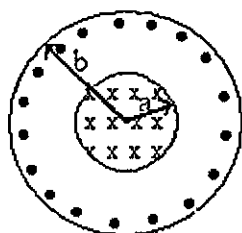


Fig.3.



4. (a) State the Divergence theorem. (2%)  
(b) Prove the Divergence theorem by the vector function  $\mathbf{A} = a_r r^2 + a_z 3z$ , enclosed in the cylindrical region of  $r = 5$ ,  $z = 0$  and  $z = 4$ . (10%)  
(c) Give an electromagnetic example which uses the Divergence theorem to solve the problem (3%)
5. A positive charge  $Q$  was placed in the center of a dielectric shell with inner radius  $R_i$  and outer radius  $R_o$ . The relative permittivity of the shell was  $\epsilon_r$ .  
(a) Determine the electric field intensity  $\mathbf{E}$ , the electric potential  $V$ , the electric displacement  $\mathbf{D}$ , and the polarization vector  $\mathbf{P}$  as the function of radial distance  $R$ . (12%)  
(b) Write down the relation between  $\mathbf{E}$ ,  $\mathbf{D}$ , and  $\mathbf{P}$ . (3%)
6. A long cylindrical conducting wire, with a radius  $a$  and conductivity  $\sigma$ , was coated on the surface with the material which has a conductivity  $0.1\sigma$ .  
(a) Calculate the thickness of the coating sheath material needed to get the whole coated wire resistance per length 50% lower? (8%)  
(b) Suppose the total current in the coated wire is  $I$ , calculate the current density  $\mathbf{J}$  and electric field intensity  $\mathbf{E}$  distributed in the core and the outer coating sheath, respectively. (12%)



1. Explain the following terms:
  - (a) Acceptors in a semiconductor (5%)
  - (b) Carrier mobility (5%)
  - (c) Fermi level in a semiconductor (5%)
  - (d) Zener tunneling (5%)
  
2. Explain the formation of the depletion region in a semiconductor p-n junction diode. (15%)
  
3. Explain the thermionic emission mechanism in a Schottky-barrier diode. (15%)
  
4. Consider a p-type silicon substrate doped to  $1 \times 10^{17} \text{ cm}^{-3}$ , and the thickness of silicon dioxide  $t_{\text{ox}} = 200 \text{ \AA}$ . ( $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ ,  $\epsilon_s = 11.7 \epsilon_0$ ,  $\epsilon_{\text{ox}} = 3.9 \epsilon_0$ ,  $\epsilon_0 = 8.85 \times 10^{-14}$  and  $E_g = 1.12 \text{ eV}$ )
  - (a) Calculate the gate voltage of an ideal MOS structure with  $n^+$  poly-silicon gate when the surface potential  $\phi_s = (3/2)\phi_{fp}$ . (8%)
  - (b) As problem (a) find the capacitance  $C/\text{cm}^2$  (6%)
  - (c) And the threshold voltage. (6%)
  
5. Draw the high- and low-frequency C-V characteristics of the n-type substrate. And indicated the correspond voltages to flat-band, weak inversion, accumulation, depletion and strong inversion mode. (15%)
  
6. Draw the minority carrier distributions for n-p-n bipolar transistor in cut off, saturation and forward active mode, respectively. (15%)