



- Find the solution of the following equation:
  - $(3xe^y + 2y)dx + (x^2e^y + x)dy = 0$ . (10%)
  - $y'' + 4y' + 4y = \frac{e^{-2x}}{x^2}$ ;  $y(1) = e^{-2}$ ,  $y'(1) = -2e^{-2}$ . (10%)
  - $x^3y''' + x^2y'' - 2xy' + 2y = x^3 \ln x$ . (15%)
  - $y'' - 16ty' + 32y = 14$ ;  $y(0) = y'(0) = 0$ . (15%)
- A copper ball is heated to a temperature of  $100^\circ\text{C}$ . Then at time  $t = 0$ , it is placed in water that is maintained at a temperature of  $30^\circ\text{C}$ . At the end of 3 minutes, the temperature of the ball is reduced to  $70^\circ\text{C}$ . Find the time at which the temperature of the ball is reduced to  $40^\circ\text{C}$ . (10%)
- Determine the frequency of oscillation of the pendulum of length  $L$  in Fig A. Neglect air resistance and the weight of the rod. Assume that  $\theta$  is small enough that  $\sin \theta \approx \theta$ . (10%)
- If vector field  $\vec{F} = x^3i + y^3j + z^3k$ , bounded surface  $S: x^2 + y^2 + z^2 = 4$ . Please evaluate the surface integral  $\iint_S \vec{F} \cdot \vec{n} \, dA$ . (15%)
- If vector field  $\vec{F} = y^3i - x^3j$ , the bounded circular disk  $S: 1 \leq x^2 + y^2 \leq 4, z = 0$ . Please evaluate  $\iint_S (\text{curl } \vec{F}) \cdot \vec{n} \, dA$ . (15%)

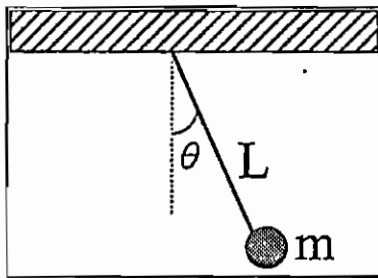
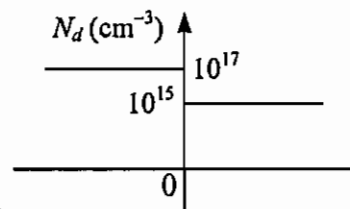


Fig A



1. Calculate the ionization energy and radius of the donor electron in silicon using the Bohr theory. (20%)
2. A silicon Hall device at  $T = 300$  K has the following geometry:  $d = 10^{-3}$  cm,  $W = 10^{-2}$  cm and  $L = 10^{-1}$  cm. The following parameters are measured;  $I_x = 0.75$  mA,  $V_x = 15$  volts,  $V_H = +5.8$  mV, and  $B_z = 1000$  gauss =  $10^{-1}$  Tesla. (20%)  
Determine (a) the conductivity type. (5%)  
(b) the majority carrier concentration (7%)  
and (c) the majority carrier mobility. (8%).
3. The doping profile of an n-n isotype step junction is shown in the figure. (14%)  
Determine the built-in potential barrier.



4. Explain the following non-ideal effects in a bipolar transistor: (a) base width modulation, and (b) current crowding. (16%)
5. The metallurgical base width of a silicon npn bipolar transistor is  $W_B = 0.6$   $\mu\text{m}$ . The base and collector doping concentrations are  $N_B = 3 \times 10^{16}$   $\text{cm}^{-3}$  and  $N_C = 10^{15}$   $\text{cm}^{-3}$ . Neglecting the space charge width of the B-E junction, find the punch-through breakdown voltage at the B-C junction. (15%)
6. An ideal n-channel MOSFET has an inversion carrier mobility  $\mu_n = 525$   $\text{cm}^2/\text{V}\cdot\text{s}$ , a threshold voltage  $V_T = +0.53$  V, and an oxide thickness  $t_{\text{ox}} = 100$   $\text{\AA}$ . When biased in the saturation region, the required rated current is  $I_D(\text{sat}) = 5$  mA when  $V_{\text{GS}} = 3$  V. Determine the required  $W/L$  ratio.  $T = 300$  K. (15%).

Table B.4 | Silicon, gallium arsenide, and germanium properties ( $T = 300$  K)

Property	Si	GaAs	Ge
Atoms ( $\text{cm}^{-3}$ )	$5.0 \times 10^{22}$	$4.42 \times 10^{22}$	$4.42 \times 10^{22}$
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density ( $\text{g}/\text{cm}^{-3}$ )	2.33	5.32	5.33
Lattice constant ( $\text{\AA}$ )	5.43	5.65	5.65
Melting point ( $^{\circ}\text{C}$ )	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, $\chi$ (volts)	4.01	4.07	4.13
Effective density of states in conduction band, $N_c$ ( $\text{cm}^{-3}$ )	$2.8 \times 10^{19}$	$4.7 \times 10^{17}$	$1.04 \times 10^{19}$
Effective density of states in valence band, $N_v$ ( $\text{cm}^{-3}$ )	$1.04 \times 10^{19}$	$7.0 \times 10^{18}$	$6.0 \times 10^{18}$
Intrinsic carrier concentration ( $\text{cm}^{-3}$ )	$1.5 \times 10^{10}$	$1.8 \times 10^6$	$2.4 \times 10^{13}$
Mobility ( $\text{cm}^2/\text{V}\cdot\text{s}$ )			
Electron, $\mu_n$	1350	8500	3900
Hole, $\mu_p$	480	400	1900
Effective mass ( $\frac{m^*}{m_0}$ )			
Electrons	$m_i^* = 0.98$ $m_t^* = 0.19$	0.067	1.64 0.082
Holes	$m_{lh}^* = 0.16$ $m_{hh}^* = 0.49$	0.082 0.45	0.044 0.28
Effective mass (density of states)			
Electrons ( $\frac{m_n^*}{m_0}$ )	1.08	0.067	0.55
Holes ( $\frac{m_p^*}{m_0}$ )	0.56	0.48	0.37



Table B.3 | Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{23}$ atoms per gram molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$e = 1.60 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s $\frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.998 \times 10^{10}$ cm/s
Thermal voltage ( $T = 300$ K)	$V_t = \frac{kT}{e} = 0.0259$ volt $kT = 0.0259$ eV

Table B.6 | Properties of  $\text{SiO}_2$  and  $\text{Si}_3\text{N}_4$  ( $T = 300$  K)

Property	$\text{SiO}_2$	$\text{Si}_3\text{N}_4$
Crystal structure	[Amorphous for most integrated circuit applications]	
Atomic or molecular density ( $\text{cm}^{-3}$ )	$2.2 \times 10^{22}$	$1.48 \times 10^{22}$
Density ( $\text{g-cm}^{-3}$ )	2.2	3.4
Energy gap	$\approx 9$ eV	4.7 eV
Dielectric constant	3.9	7.5
Melting point ( $^\circ\text{C}$ )	$\approx 1700$	$\approx 1900$



1. Given a vector function  $\mathbf{F} = a_x(x+c_1z) + a_y(c_2x-3z) + a_z(x+c_3y+c_4z)$ . Determine the constants  $c_1$ ,  $c_2$ ,  $c_3$ , and  $c_4$ , if  $\mathbf{F}$  is irrotational and also solenoidal. (15%)
2. The two parallel conducting wires of a power transmission line have a radius  $a$  and are spaced at a distance  $d$  apart. The wires are at a height  $h$  above the ground. Assuming the ground to be perfectly conducting and both  $d$  and  $h$  to be much larger than  $a$ , find the expression for the mutual and self-partial capacitances per unit length. (15%)
3. Consider a plane boundary ( $y=0$ ) between air (region 1,  $\mu_{r1}=1$ ) and iron (region 2,  $\mu_{r2}=5000$ ). Assuming  $\mathbf{B}_1 = a_x 0.5 - a_y 10$  (mT), find  $\mathbf{B}_2$  and the angle that  $\mathbf{B}_2$  makes with the interface. (20%)
4. The maximum electric field at a distance of 10 m from an isotropic point light source is 2.0 V/m. What are (a) the maximum value of the magnetic field and (b) the average intensity of the light there? (c) What is the power of the source? (15%)
5. A plane electromagnetic wave, with wavelength 3.0 m, travels in vacuum in the positive direction of an  $x$  axis. The electric field, of amplitude 300 V/m, oscillates parallel to the  $y$  axis. What are the (a) frequency? (b) angular frequency, and (c) angular wave number of the wave? (d) What is the amplitude of the magnetic field component? (e) What is the time averaged rate of energy flow in watts per square meter associated with this wave? The wave uniformly illuminates a surface of area  $2.0 \text{ m}^2$ . (25%)
6. The induced magnetic field 6.0 mm from the central axis of a circular parallel-plate capacitor and between the plates is  $2.0 \times 10^{-7}$  T. The plates have radius 3.0 mm. At what rate  $d\bar{E}/dt$  is the electric field between the plates changing? (10%)