



1. (15%) Given any positive integers  $a$  and  $b$ . Prove that there exists a unique positive integer  $c$  that is the greatest common divisor of  $a$  and  $b$ .
2. (10%) Let  $R[x]$  be the set of all polynomials in the indeterminate  $x$  with coefficients from  $R$ . Given that  $R$  is a ring. Prove that under the operations of polynomial addition (+) and multiplication (\*),  $(R[x], +, *)$  is a ring.
3. (a) (5%) If  $A$  is a set, then prove any equivalence relation  $R$  on  $A$  induces a partition of  $A$ .  
 (b) (5%) If  $A$  is a set, then prove that any partition of  $A$  gives rise to an equivalence relation  $R$  on  $A$ .
4. (15%) Define the Fibonacci numbers as follows:  
 $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$ , for  $n \geq 2$ .  
 Prove that any two consecutive Fibonacci numbers are relatively prime.
5. (10%) Which of the following are statements?  
 (a) The moon is made of green cheese.  
 (b) He is certainly a tall man.  
 (c) Two is a prime number.  
 (d) Will the game be over soon?  
 (e) Next year interest rates will rise.
6. (10%) Prove that for any positive integer  $n$ , the number  $2^{2^n} - 1$  is divisible by 3.
7. (8%) (a) How many distinct permutations are there of the characters in the word HAWAIIAN? (b) How many of these must begin with H?
8. (12%) Consider the following Turing machine:
 

(0, 0, 0, 1, R)
(0, 1, 0, 0, R)
(0, b, b, 0, R)
(1, 0, 1, 0, R)
(1, 1, 1, 0, R)

  - (a) What is the final tape, given the initial tape  $\overline{\dots b1|0b\dots}$  ?
  - (b) Describe the behavior of the machine when started on the tape  $\overline{\dots b0|1b\dots}$  .
  - (c) Describe the behavior of the machine when started on the tape  $\overline{\dots b0|0b\dots}$  .
9. (10%) Let  $\rho$  be a binary relation on a set  $S$ . Then a binary relation called the *inverse* of  $\rho$ , denoted  $\rho^{-1}$ , is defined by  $x \rho^{-1} y \leftrightarrow y \rho x$ . Prove that if a binary relation  $\rho$  on a set  $S$  is reflexive and transitive, then the relation  $\rho \cap \rho^{-1}$  is an equivalence relation.



題目1至題目10為多選題，每題5分。每題需全部答對才給分，答錯倒扣1分。

1. What storage systems are volatile?
  - (A) registers
  - (B) cache
  - (C) flash memory
  - (D) magnetic disk
  
2. Which are correct for the message-passing model and shared-memory model of interprocess communication?
  - (A) Message passing is useful for exchanging smaller amounts of data.
  - (B) Message passing is harder to implement than is shared memory for intercomputer communication.
  - (C) Shared memory allows maximum speed and convenience of communication.
  - (D) Problems exist in the areas of protection and synchronization between the processes sharing memory.
  
3. What CPU scheduling can be either preemptive or nonpreemptive?
  - (A) FCFS scheduling
  - (B) SJF scheduling
  - (C) Priority scheduling
  - (D) Round-Robin scheduling
  
4. Which are correct for deadlocks?
  - (A) Deadlock prevention provides methods for ensuring that at least one of necessary conditions for a deadlock situation cannot hold.
  - (B) For deadlock avoidance, if a process request a resources that is currently available, it may still have to wait.
  - (C) Preventing deadlocks by eliminating the hold-and-wait condition may incur low resource utilization.
  - (D) Semaphores can be used to solve deadlocks.



5. Which are correct for a paging system?
- (A) No fragmentation problem is incurred when using a paging scheme.
  - (B) Using a translation look-aside buffer, only one memory access is required during the page-address translation.
  - (C) For the shared pages in multiple processes, the relative positions in the page table for each process should be the same.
  - (D) In the system using an inverted page table, only one page table is required
6. Which are correct for page replacement?
- (A) FIFO page replacement may incur Belady's anomaly.
  - (B) The optimal page replacement algorithm is easy to implement.
  - (C) Both optimal and LRU page replacement are called stack algorithms.
  - (D) The LFU page replacement algorithm is based on the argument that the page with the smallest count was probably just brought and has yet to be used.
7. Which are correct for directory structure?
- (A) The directory can be viewed as a symbol table that translates file names into their directory entries.
  - (B) The implementation for the shared and subdirectories in many UNIX systems is to create a new directory entry called a link.
  - (C) The deletion of a link implies the file deletion.
  - (D) In a general graph directory, cycles are allowed to exist in the directory.
8. Which are correct for disk space allocation?
- (A) For contiguous allocation, the size of a file need not be declared when that file is created.
  - (B) Contiguous allocation suffers from the external fragmentation problem.
  - (C) Linked allocation is inefficient to support a direct access capability.
  - (D) Indexed allocation supports direct access without suffering from external fragmentation.
9. What disk scheduling may cause starvation of some requests?
- (A) FCFS scheduling
  - (B) SSTF scheduling
  - (C) SCAN scheduling
  - (D) LOOK scheduling



10. Which are correct for a storage-area network (i.e., SAN)?
- (A) a private network
  - (B) using storage protocols rather than networking protocols
  - (C) Storage can be dynamically allocated to hosts.
  - (D) A fiber channel is the most common SAN interconnect.

題目11至題目20為簡答題，每題5分。

11. List two advantages of paging.
12. List two ways of treating jobs which request too much memory, once started in a given partition.
13. List three examples of real-time systems.
14. List three resources must be protected by the operating system.
15. Describe three methods for passing parameters needed by system calls.
16. What is a context switch?
17. List three ways of allocating storage.
18. Consider a logical address space of eight pages of 1024 words each, mapped onto a physical memory of 32 frames.
- a. How many bits are there in the logical address?
  - b. How many bits are there in the physical address?
19. What three requirements must a solution to the critical-section problem satisfy?
20. What are the four necessary conditions needed before the deadlock occur?



1. If  $x_1, x_2, \dots, x_n$  are numbers, then show by induction that

$$\begin{vmatrix} 1 & x_1 & \dots & x_1^{n-1} \\ 1 & x_2 & \dots & x_2^{n-1} \\ \dots & \dots & \dots & \dots \\ 1 & x_n & \dots & x_n^{n-1} \end{vmatrix} = \prod_{i < j} (x_j - x_i),$$

the symbol on the right meaning that it is the product of all terms  $x_j - x_i$  with  $i < j$  and  $i, j$  integers from 1 to  $n$ . (10%)

2. Determine the sign of the following permutations. At the same time, write the inverse of the permutation. (10%)

(a)  $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ ; (b)  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{bmatrix}$ .

3. Compute the eigenvalues of the following matrix in complex numbers. (10%)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

4. Let  $X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $X_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ , and  $X_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ . Please get an orthonormal base for  $\{X_1, X_2, X_3\}$ . (10%)

5. Let  $A = \begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & 3 \\ 4 & -1 & 1 \end{bmatrix}$ . Please get the adjoint of  $A$  ( $adjA$ ). (Note that if  $A$  is a  $n \times n$  square matrix, then  $A \bullet adjA = adjA \bullet A = \det A \bullet I_n$ .) (10%)

6. Give 5 statements that are equivalent to a square  $n \times n$  matrix being invertible. (10%)



7. Let a transformation  $T$

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} 2x_2 - x_1 - 2x_3 \\ x_1 - x_2 \end{bmatrix}$$

- (a) (5%) Show that  $T$  is a linear transformation.  
 (b) (5%) Find the transformation matrix of  $T$  with respect to the standard basis.  
 (c) (10%) Find an orthonormal basis for the kernel of the matrix of  $T$ ,  $\ker(T)$ . Give the orthogonal decomposition of the vector  $[1 \ 1 \ -1]^T$  with respect to  $\ker(T)$ .

8. Suppose  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$  are linearly independent vectors. Show that  $\mathbf{u}$ ,  $\mathbf{u} + \mathbf{v}$ , and  $\mathbf{u} + \mathbf{v} + \mathbf{w}$  are linearly independent. (10%)

9. Let  $T$  is a transformation from  $\mathbf{R}^2$  to  $\mathbf{R}^2$  and

$$T \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 \cos \theta - x_2 \sin \theta \\ x_1 \sin \theta + x_2 \cos \theta \end{bmatrix} = \mathbf{A}\mathbf{x}$$

- (a) (5%) Find the inverse transformation of  $T$ .  
 (b) (5%) Use the inverse transformation found in (a) to solve the system given by

$$\mathbf{A}\mathbf{x} = \mathbf{b} \text{ where } \mathbf{b} = [1 \ -1]^T \text{ and } \theta = \pi.$$



## 一、填充題 (每題一分)

1. \_\_\_\_\_ is an implementation technique in which multiple instructions are overlapped in execution.
2. \_\_\_\_\_ is used to hold the address of the next instruction.
3. The term \_\_\_\_\_ to refer to any unexpected change in control flow without distinguishing whether the cause is internal or external.
4. The term \_\_\_\_\_ to refer to any unexpected change in control flow only cause in external.
5. \_\_\_\_\_ means that the hardware cannot support the combination of instructions that we want to execute in the same clock cycle.
6. \_\_\_\_\_ occur when the pipeline must be stalled because one step must wait for another to complete.
7. The \_\_\_\_\_ scheme always writes the data into both the memory and the cache.
8. The \_\_\_\_\_ consists of multiple levels of memory with different speeds and sizes.
9. The \_\_\_\_\_ is the fraction of memory accesses found in the upper level.
10. In a \_\_\_\_\_ cache, all the entries in the cache must be searched.

## 二、選擇題(每題二分)

1. An eight-way set-associative cache in a computer in which the real memory size is  $2^{32}$  bytes. The block size is 16 bytes, and there are  $2^{10}$  set. Which of the following statement is incorrect? (A) the tag is 16 bits, (B) the tag is 18 bits, (C) the tag is 20 bits, (D) the tag is 10 bits
2. A digital computer has a main memory of 32MB and a fully associative cache memory of 512KB. The cache block is 64 byte. How many bits are there in the tag field? (A)6 (B)13 (C)19 (D)25.
3. Continued with question 12, if a direct mapping scheme were used instead, what would be the size of the tag field? (A)6 (B)13 (C)19 (D)25
4. Assume that each instruction occupies 2 bytes. A PC-relative mode branch instruction is stored in memory at address  $600_{10}$ . The branch is made to location  $520_{10}$ . The address field in the instruction is 10 bits long. What is the binary value in the instruction? (A)10101110 (B)01010010 (C)01010011 (D)10101101
5. Convert  $-43\frac{7}{32}$  to IEEE 754 32-bit floating point format. Which of the following statement is incorrect? (A) bit 31 is 1 (B) bit 22 is 1 (C) the exponent part is 132 (D) bit 0 is 0.
6. Assume that an instruction pipeline has five stages with propagation delays of 20ns, 25ns, 20ns, 70ns, and 40 ns, respectively. Which of the following statement is correct? (A) the clock period of the pipeline is 75 (B) the pipeline need 7.28 ms to complete 100 instructions (C) the pipeline need 3.64 ms to complete 50 instructions (D) the speedup of the pipeline is 2.4



- [ ] 7. \_\_\_\_ circuits are used to determine the order in which operations are carried out inside a computer and to select the correct data values to be processed. (A) sequential (B) combinational (C) control (D) order
- [ ] 8. If the last operation performed on a computer with an 8-bit word was an addition in which the two operands were 0000010 and 0000011. Which of the following statement is incorrect? (A) Carry bit equal to 0, (B) Zero bit equal to 0 (C) Overflow bit equal to 0 (D) Even parity bit equal to 0
- [ ] 9. Consider a 16-bit processor in which the following appears in main memory, starting at location 200:

200	Load to AC	Mode
201	500	
202	Next instruction	

The first part of the first word indicates that this instruction loads a value into an accumulator. The Mode field specifies an addressing mode and, if appropriate, indicates a source register; assume that when used, the source register is R1 which has a value of 400. There is also a base register that contains the value 100. The value of 500 in location 201 may be part of the address calculation. Assume that location 500 contains the value 1100. Which of the following statement is incorrect? (A) the EA of indirect addressing is 1100, (B) the EA of register addressing is 400, (C) the operand of direct addressing is 1100, (D) the operand of PC relative addressing is 702.

- [ ] 10. Same as question 19, Which of the following statement is correct? (A) the EA of direct addressing is 500, (B) the EA of displacement addressing is 700, (C) the EA of register indirect addressing is 402, (D) the EA of PC relative addressing is 1302.

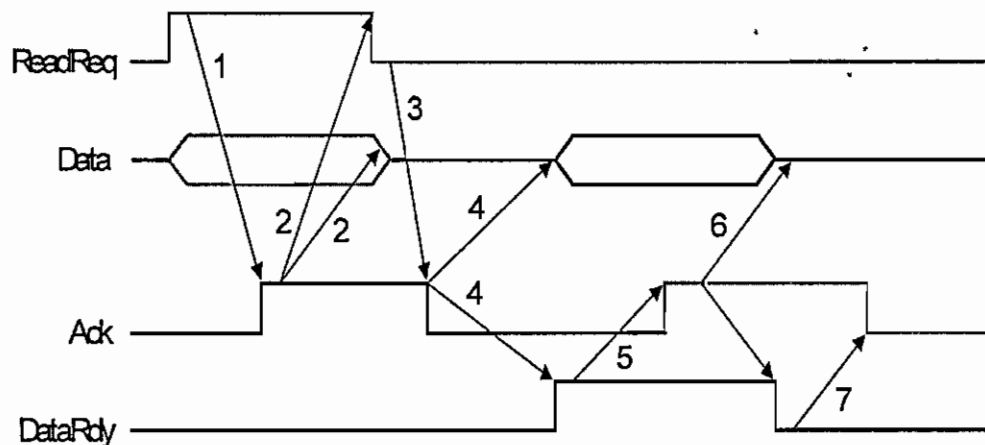
### 三、問答計算題

1. (10 points) Here is a series of address references given as word addresses: 1, 4, 8, 5, 20, 17, 19, 56, 9, 11, 4, 43, 5, 6, 9, 17.
- (a) Assuming a direct-mapped cache with 16 one-word blocks that is initially empty, label each reference in the list as a hit or miss, show the final contents of the cache.
- (b) Show the hits or misses and final cache contents for a direct-mapped cache with four-word blocks and a total size of 16 words.
- (c) Show the hits or misses and final cache contents for a two-way set associative cache with one-word blocks and a total size of 16 words. Assume LRU replacement.
- (d) What are the hit ratios of above question?





2. (5 points) The asynchronous handshaking protocol consists of seven steps to read a word from memory and receive it in an I/O device. Please depict these steps briefly.



3. (5 points) (a) Assume that multiply instructions take 12 cycles and account for 15% of the instructions in a typical program, and the other 85% of the instructions require an average of 4 cycles for each instruction. What percentage of time does the CPU spend doing multiplication?
- (b) Your hardware engineering team has indicated that it would be possible to reduce the number of cycles required for multiplication to 8, but this will require a 20% increase in the cycle time. Nothing else will be affected by the change. Should they proceed with the modification?
4. (12 points) How many total bits are required for a direct-mapped cache with 16KB of data and one-word blocks, assuming a 32-bit address? If the CPU wants to read memory address 0xABCDABCD, what is the tag value of this address?
5. (12 points) In the cache write operations, what are the *write-through* policy and *write-back* policy? What is the purpose of the *write buffer*?
6. (10 points) For pipelined execution, assume that half of the load instructions incur the data hazards, that the one-quarter of the branches have control hazards. Assume all other instructions have a CPI of 1. If one program has 30% loads, 15% stores, 20% branches, 5% jumps, and 30% ALU. What is the average CPI?
7. (8 points) Suppose we have two implementations of the same instruction set architecture. Machine A has a clock cycle time of 1 ns and a CPI of 2.0 for some program, and machine B has a clock cycle time of 2 ns and a CPI of 1.2 for the same program. Which machine is faster for this program, and by how much?
8. (8 points) Consider a program consisting of 500 lw instructions and in which each instruction is independent upon the instruction before it. What would the actual CPI be if the program were run on the 6-stage pipelined datapath?



## 一、選擇題 (題目 1 到題目 20，每題一分。題目 21 到題目 30，每題二分。 )

- [ ] 1. The operation "If  $x = 0$ , then  $y = 0$ , otherwise let  $y = 2x$ " is a(n) \_\_\_\_\_ operation.  
 (A) hierarchal (B) iterative (C) sequential (D) conditional
- [ ] 2. Which of the following is an unambiguous, effectively computable operation?  
 (A) repeat the next few operations 10 times (B) see whether or not  $x$  equals  $y$  (C) list the positive integers (D) add  $x$  and  $y$ , repeat
- [ ] 3. A(n) \_\_\_\_\_ is a well-ordered collection of unambiguous and effectively computable operations that, when executed, produces a result and halts in a finite amount of time.  
 (A) sequence (B) computing agent (C) mechanical calculator (D) algorithm
- [ ] 4. Most computer scientists use \_\_\_\_\_ to design and represent algorithms.  
 (A) natural languages (B) high-level programming languages (C) low-level programming languages (D) pseudocode
- [ ] 5. A(n) \_\_\_\_\_ is a named storage location that can hold a data value.  
 (A) expression (B) variable (C) computation (D) constant
- [ ] 6. Together, conditional and iterative operations are called \_\_\_\_\_ operations.  
 (A) sequential (B) control (C) hierarchical (D) dynamic
- [ ] 7. In an infinite loop, the \_\_\_\_\_.  
 (A) continuation condition never becomes false (B) continuation condition never becomes true  
 (C) loop body never becomes false (D) loop body never becomes true
- [ ] 8. In a pretest loop, the continuation condition is tested at the \_\_\_\_\_ through the loop.  
 (A) beginning of each pass (B) beginning of only the first pass (C) end of each pass (D) end of only the last pass
- [ ] 9. A posttest loop is expressed through a(n) \_\_\_\_\_ statement.  
 (A) if/then/else (B) While (C) Do/While (D) if
- [ ] 10. The technique of looking at all the items in a list, one at a time, until we either find what we are looking for or come to the end of the list is called \_\_\_\_\_ search.  
 (A) sequential (B) binary (C) iterative (D) random
- [ ] 11. \_\_\_\_\_ means the ability to separate the high-level view of an entity or an operation from the low-level details of its implementation. (A) inheritance (B) randomness (C) encapsulation (D) abstraction



- [ ] 12. Once a program is written, fixing any errors that are uncovered through repeated usage with different input values and extending the program to meet new requirements is called program \_\_\_\_\_.  
 (A) networking (B) service (C) maintenance (D) development
- [ ] 13. The study of the efficiency of various algorithms is called the \_\_\_\_\_ of algorithms.  
 (A) design (B) analysis (C) implementation (D) testing
- [ ] 14. Sequential search is a(n) \_\_\_\_\_ algorithm in the worst case.  
 (A)  $O(n)$  (B)  $O(1)$  (C)  $O(2^n)$  (D)  $O(n^2)$
- [ ] 15. The binary search algorithm is \_\_\_\_\_ in the best case. (A)  $O(1)$  (B)  $O(n)$  (C)  $O(n^2)$  (D)  $O(\log n)$
- [ ] 16. In the \_\_\_\_\_ phase, the compiler examines the individual characters in the source program and groups them into tokens. (A) semantic analysis and code generation (B) parsing (C) lexical analysis (D) code optimization
- [ ] 17. In the \_\_\_\_\_ phase, the compiler analyzes the meaning of the tokens making up a programming language statement and tries to understand the actions they perform. (A) semantic analysis and code generation (B) parsing (C) lexical analysis (D) code optimization
- [ ] 18. Logic programming languages are also called \_\_\_\_\_ languages. (A) declarative (B) applicative (C) descriptive (D) imperative
- [ ] 19. In BNF, a \_\_\_\_\_ is an intermediate grammatical category used to help explain and organize the language. (A) production (B) rule (C) terminal (D) nonterminal
- [ ] 20. The \_\_\_\_\_ of a disk is the time needed to position the read/write head over the correct track. (A) latency time, (B) position time, (C) data transfer time, (D) seek time.
- [ ] 21. Given the sequence (C, O, M, P, U, T, E, R) to construct a binary search tree. Which of the following statement is incorrect? (A) E is a leaf, (B) P is an internal node, (C) T is a leaf, (D) R is a leaf.
- [ ] 22. Given the sequence (C, O, M, P, U, T, E, R) to construct a AVL tree. Which of the following statement is incorrect? (A) P is the root, (B) U is an internal node, (C) R is a leaf, (D) O is a leaf.
- [ ] 23. Assume that the preorder and inorder traversal of a binary tree is ABCDEFG and CBDAEGF, respectively. Which of the following statement is incorrect? (A) both C and D are children of node B, (B) node G is a leaf, (C) A is the root, (D) F is a right-children of node E
- [ ] 24. Which of the following prefix notation is incorrect? (A)  $+ab-cd-e$ , (B)  $+5x^2+abcd$ , (C)  $---ab+cde$ , (D)  $+xxabcde$ .
- [ ] 25. At about what value of  $n$  does an algorithm that does  $100n^2$  instructions become more efficient than



one that does  $0.01(2^n)$  instruction? (A)20, (B)21, (C)22, (D)23

- [ ] 26. Consider a three dimensional array  $A(1..4, 1..4, 1..6)$  in row-major order, and each element occupies one byte. The location of  $A(1,2,3)$  is 126. What is the location of  $A(2,1,2)$ ? (A)143, (B)144, (C)145, (D)146
- [ ] 27. What is the time complexity of the following algorithm? (A)  $O(n^2)$  (B)  $O(n)$  (C)  $O(n \log n)$  (D)  $O(2^n)$
- ```

int i, j, x[N][N], y[N][N];
for (i = 0; i < n; i++)
  for (j = i+1; j < 50; j++)
    y[i][j] = x[i][j]/2;
  
```
- [ ] 28. What is the inorder traversal of Fig. 1. (A)ABCDEFGHI (B)EFBCGHIDA (C)EBFACGDHI (D)EBFCAGHDI
- [ ] 29. Given the program segment listed as Fig. 2, if the starting address of  $a[]$  is 200 and an integer is stored with 2 bytes in memory. What is the value of  $p2$  after the "for" loop? (A)210, (B)75, (C)200, (D)100
- [ ] 30. Same as the question 29, What is the value of  $a[5]$  after the "for" loop? (A)210, (B)75, (C)80, (D)85

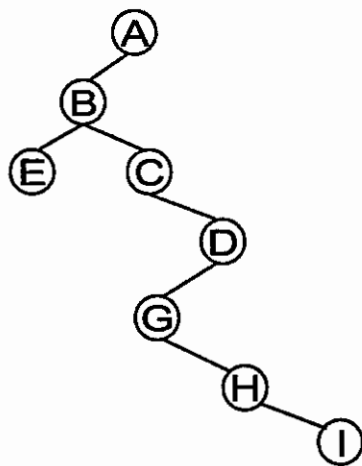


Fig.1. binary tree for question 28.

```

#include <stdio.h >
main () {
  int a[10], *p1, *p2, b=100, i;
  p1=a;
  p2=&a[5];
  for (i=0; i<10; i++) {
    *p1=b, p1++, b-=5;
  }
}
  
```

Fig.2. program segment for questions 29 and 30.

## 二、簡答題：

- (10 points) Suppose the pattern-matching problem is changed to require locating only the first instance, if any, of the pattern within the text.
  - Describe the worst case, given an example, and give the exact number of comparisons (of a pattern character with a text character) required.
  - Describe the best case, given an example, and give the exact number of comparisons required.



2. (10 points) Using the *digit-extraction* method (first, and fifth digits) for hash search and *quadratic probing* for collision resolution, store the keys shown below in an array with  $n$  elements. Please figure out the final address of every key. What is the density of the list after all keys have been inserted?  
24562, 27453, 24562, 40145, 24572, 32146, 44467, 49645, 34534
3. (10 points) Transfer the following *infix* expression into a *postfix* expression. Use the *stack* technique to complete this transformation. Please show the procedure-- a total of 15 steps, step-by-step clearly.  
 $A*B-(C+D)+E$
4. (10 points) A tennis tournament has  $n$  players, where  $n > 1$ . A single match involves two players. The winner of a match will play the winner of a match in the next round, where losers are eliminated from the tournament. The two players who have won all previous rounds play the final game, and the winner wins the tournament. What is the total number of matches needed to determine the winner? Write a C++ function, *int NumberOfMatches ( int numberOfPlyers )*, to perform this task.
5. (10 points) What are object-oriented programming's benefits comparing with traditional programming's?
6. (10 points) Describe the differences between a modern database system and a traditional file-oriented system.