國立雲林科技大學 108 學年度 碩士班招生考試試題

系所：電機系

科目：工程數學(2)

1. (15%) Solve the general solution of the following differential equations: [解下列微分方程式之通解]

   (1) $y' = 10\cos 5x$ (5 分); (2) $y' + 2xy = 2xe^{-x^2}$ (5 分); (3) $y' = \dfrac{y-2x}{x}$ (5 分).

2. (10%) Solve the following initial value problem [解下列微分方程式之解]: $y'' + 4y = 6\sin x + 8$; $y(\pi) = 2, y(\pi/4) = \sqrt{2}$ (10 分).

3. (15%) Find the Laplace transform of the following functions [求下列函數之拉氏轉換; $F(s) = L[f(t)]$]:

   (1) $f(t) = (t^2+1) - e^{-t} + \sin 2t$ (5 分);

   (2) $f(t) = \begin{cases} 1 & 0 \le t < \pi \\ 0 & \pi \le t < 2\pi \end{cases}$; $f(t+2\pi) = f(t)$ (5 分);

   (3) $f(t) = U(t-1) + \int_0^t t\sin t\, dt$ (5 分).

4. (10%) Use the Laplace transform to solve the given system equations [利用拉氏轉換求解以下聯立方程式系統]: $\dfrac{dx}{dt} + \int_0^t y dt = \delta(t)$; $x(0) = 0$ (10 分).

$$-4x + y = 0$$

5. (15%) The vector set S is $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix} \right\}$:

   (1) Determine whether the set of vector is linearly independent or linear dependent (10 分); (2) find the rank of Set S (5 分);

6. (15%) Assume matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$, and $|A| = -5$. Find

   (1) $|3A|$ (3 分); (2) $|A^{-1}|$ (3 分); (3) $|2A^{-1}|$ (3 分);

   (4) $|(2A)^{-1}|$ (3 分); (5) $\begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$ (3 分)

7. (20%) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$. Find (1) the characteristic polynomial (5 分);

   (2) eignevalues and associated eignevectors (10 分);

   (3) Find the transformation matrix P and diagonal matrix D such that $P^{-1}AP = D$ (5 分).