本試題共 7 題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. In each of problems 1 through 2. Find the general solution. (10%)

   (1) $y' = -\dfrac{2xy^3 + 2}{3x^2y^2 + 8e^{4y}}$;   (2) $x^2y'' - 5xy' + 9y = 0$

2. In each of problems 1 through 2. Find the general solution with initial value. (20%)

   (1) $y' = 3x^2 - \dfrac{y}{x}$, $y(1) = 5$;   (2) $y'' - 4y' + 53y = 0$, $y(\pi) = -3$, $y'(\pi) = 2$

3. In each of problems 1 through 2. Use the Laplace transform to solve the initial value problem. (20%)

   (1) $y'' + 4y' + 3y = e^t$; $y(0) = 0, y'(0) = 2$

   (2) $y'' - 2y' - 8y = f(t)$; $y(0) = 1, y'(0) = 0$

4. Vectors $\vec{A} = 8\hat{x} - 6\hat{y} + 4\hat{z}$, $\vec{B} = \hat{x} + \hat{y}$, $\vec{C} = \hat{x} - \hat{y}$: (1) find $\vec{A} \bullet (\vec{B} \times \vec{C})$,   (2) find the equation of the plane containing the point (2, -3, 4) and orthogonal to the vector $\vec{A}$. (10%)

5. Let $A = \begin{bmatrix} 4 & 6 & -3 \\ 2 & 3 & -4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ -7 \end{bmatrix}$;   (1) find the reduced form of $[A \vdots B]$; $[A \vdots B]_R = ?$,

   (2) find the inverse matrix of A; $A^{-1} = ?$, (3) use an inverse matrix to solve the system equation $AX = B$, (4) use Cramer's rule to solve the system equation $AX = B$.   (20%)

6. $f(x) = \begin{cases} -1, & -\pi \leq x < 0 \\ 2, & 0 \leq x < \pi \end{cases}$, find the Fourier series of $f(x)$ on the given interval.   (10%)

7. Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}$; $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$,   (1) find eigenvalues and eigenvectors of A, (2) find the general solution of $X' = AX$.   (10%)