



1. Please find the general solution of the following ordinary differential equations, where  $y$  is a function of  $x$  :

(a) (15%)  $y'' + 4y' + 4y = e^{-2x}$

(b) (15%)  $xy' + y = xe^x + 3x^2$

2. (20%) Consider the 2<sup>nd</sup> order O.D.E.

$$y'' + 4y = f(t), \quad y(0) = y'(0) = 1$$

$y$  is a function of  $t$ . Please find the initial value problems for all  $t \geq 0$  according to the  $f(t)$  defined as follows.

$$f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1 \\ 0 & \text{for } t \geq 1 \end{cases}$$



3. A line in 3-dimensional space  $R^3$  is represented by a position vector  $\vec{p}$  in  $R^3$  and given as  $\vec{p} = t\vec{v}$ ,

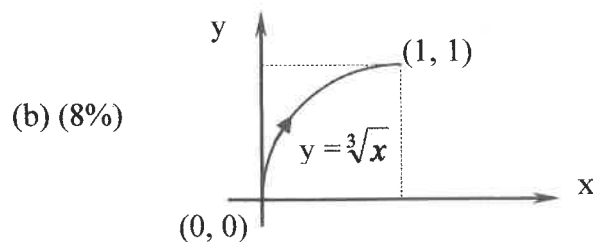
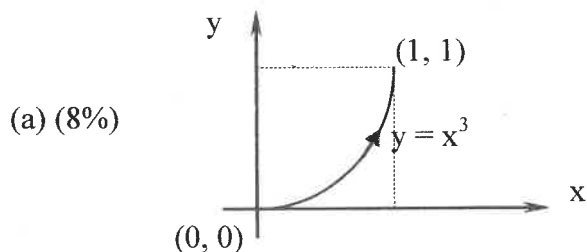
where  $t \in R$  and  $\vec{v} \in R^3$ .

A linear transformation defined by the projection of  $\vec{x}$  onto the line given above.

- (10%) Please define the linear transformation in vector form.
- (15%) Please find the eigenvalues and the corresponding eigenvectors of the linear transformation.

4. Given  $\vec{F}(t) = -2y\vec{i} + (5y - 2x)\vec{j}$  and  $\vec{r}(t) = x\vec{i} + y\vec{j}$ , please compute the line integral

$\int_C \vec{F} \cdot d\vec{r}$  with C as the following paths.



(c) (9%) Please show the reason why they have the same results.

(Hint: **Try to find the potential function**  $\phi(x,y)$  which  $\vec{F}$  is derived from.)



1. (25%) A brass bar of length 2.25 m with a square cross section of 90 mm on each side is subjected to an axial tensile force of 1500 kN (see Figure 1). Assume that Young's modulus  $E = 110$  GPa and Poisson's ratio  $\nu = 0.34$ . Determine the increase in volume of the bar.

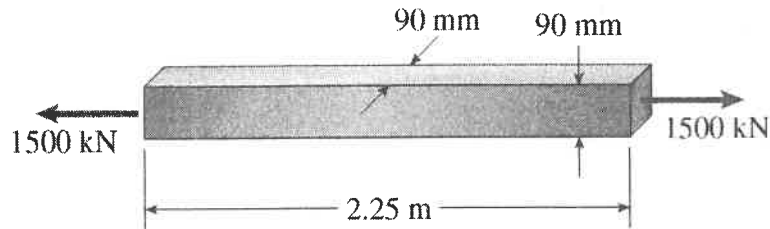


Figure 1

2. (25%) In Figure 2, the center post  $B$  of the assembly has an original length of 124.7 mm, whereas posts  $A$  and  $C$  have an original length of 125 mm. If the caps on the top and bottom can be considered rigid, determine the average normal stress in each post. The posts are made of aluminum and have a cross-sectional area of  $400 \text{ mm}^2$ . The Young's modulus  $E = 70$  GPa.

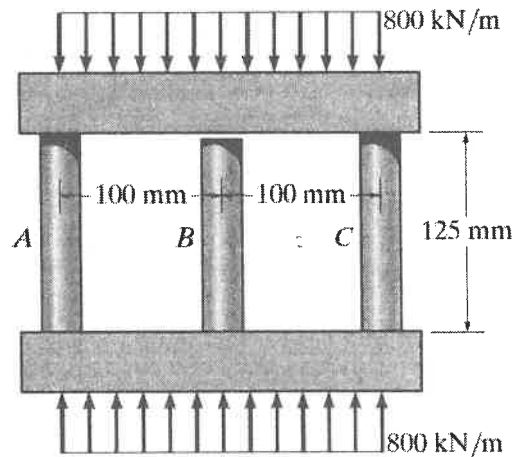


Figure 2



3. (20%) A shaft of radius  $r=10$  mm is made from an aluminum alloy and is fixed at its ends A and B. If it is subjected to the two torques shown in Figure 3, determine the reaction at the fixed supports A and B.

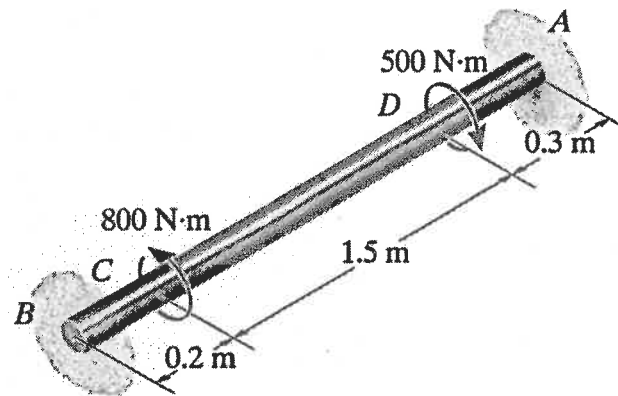


Figure 3

4. A simply supported beam in Figure 4a has the cross-sectional area shown in Figure 4b.
- (10%) Draw the moment diagram for the beam.
  - (10%) Determine the absolute maximum bending stress in the beam.
  - (10%) Draw the stress distribution over the cross section at the location for maximum bending stress.

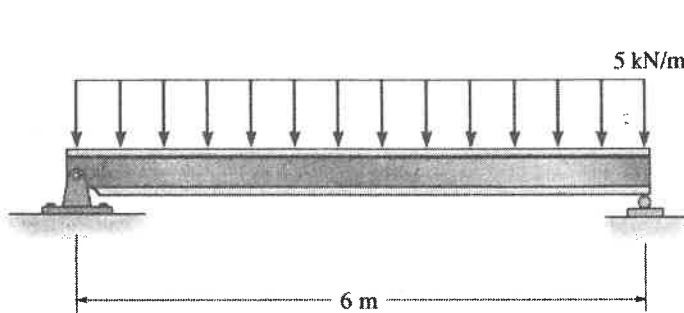


Figure 4a

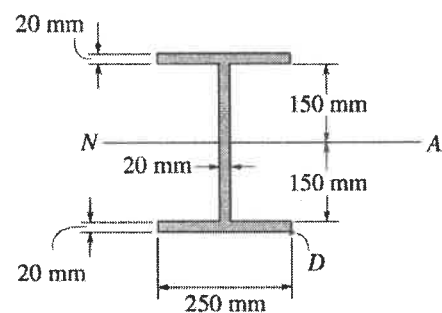


Figure 4b