



1. (25%) Please find the general solution of the following ordinary differential equation (O.D.E.):

$$y' = \frac{y + x}{x}$$

2. Consider the 2<sup>nd</sup> order O.D.E.

$$y'' + 2y' + y = f(t), \quad y(0) = y'(0) = 0$$

$y$  is a function of  $t$ . Please find the initial value problems for all  $t \geq 0$  according to each  $f(t)$  as follows.

(a)  $f(t) = 1$  for all  $t \geq 0$  (10%)

(b)  $f(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$  (15%)



國立雲林科技大學 107 學年度  
碩士班招生考試試題

系所：機械系  
科目：工程數學(1)

Prob. 3 (10%)

Find the directional derivative of  $f(x,y,z) = \sin(x - y + 2z)$   
at  $(3.2, -1.1, -3.0)$  in the direction of  $-\vec{i} + \vec{j} + \vec{k}$

Prob. 4 (15%)

Find an equation of the tangent plane to the surface of  
 $2x - \cos(xyz) = 3$  at  $(1, \pi, 1)$ .

Prob. 5 (25%)

Calculate the work done by the force  $\vec{F}(t) = \vec{i} - x\vec{j} + \vec{k}$  in moving a particle  
from  $(1,0,1)$  to  $(-1,0,\pi)$  along the curve  $x = \cos(t)$ ,  $y = \sin(t)$ ,  $z = t$ .



1. As shown in Figure 1, determine the average normal stress at section  $a-a$  and average shear stress at section  $b-b$  in member  $AB$ . The cross section of member  $AB$  is square, 0.5 in. on each side. (25%)

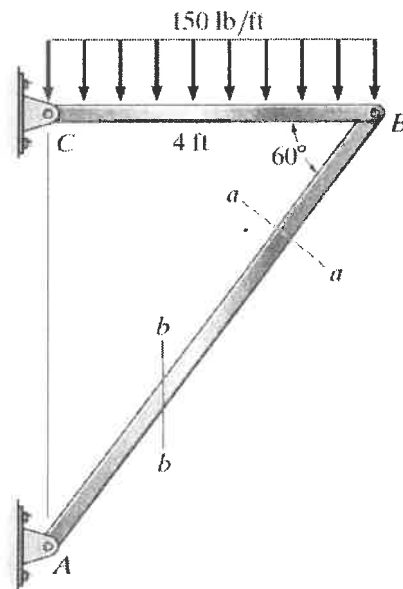


Figure 1

2. In Figure 2, the assembly consists of two copper rods  $AB$  and  $CD$  of diameter 30 mm, a stainless steel rod  $EF$  of diameter 40 mm, and a rigid cap  $G$ . The Young's modulus of copper is  $E_{copper} = 101$  GPa, and the young modulus of stainless steel alloy is  $E_{steel} = 193$  GPa. If the support  $A$ ,  $C$  and  $F$  are rigid, determine the average normal stress developed in rods  $AB$ ,  $CD$ , and  $EF$ . (25%)

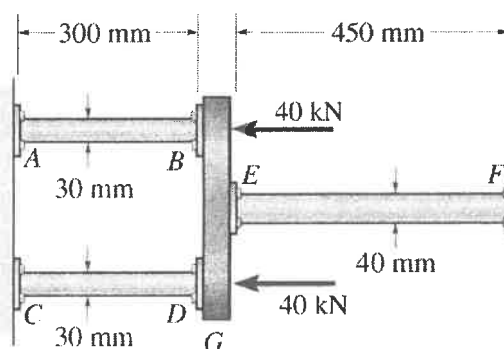


Figure 2



3. The 80-mm-diameter solid shaft is subjected to the  $1000 \text{ N}\cdot\text{m}/\text{m}$  distributed loading and  $600 \text{ N}\cdot\text{m}$  concentrated torsional loading as shown in Fig. 3. Determine the absolute maximum shear stresses in the shaft. (25%)

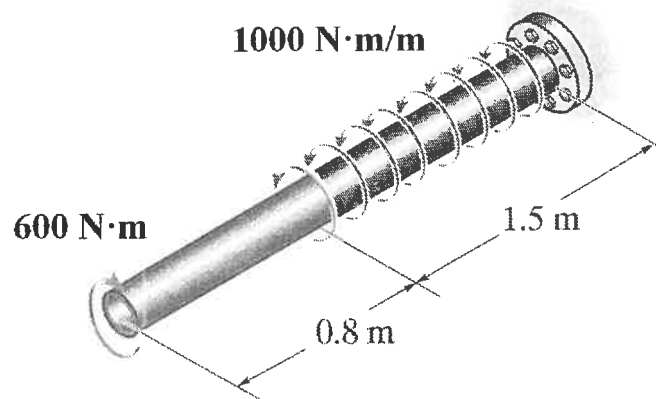


Fig. 3

4. A steel wide-flange beam has the dimensions shown in Fig. 4. Determine the maximum shear stress in the beam if it is subjected to a shear force of  $V = 10 \text{ kN}$ . (25%)

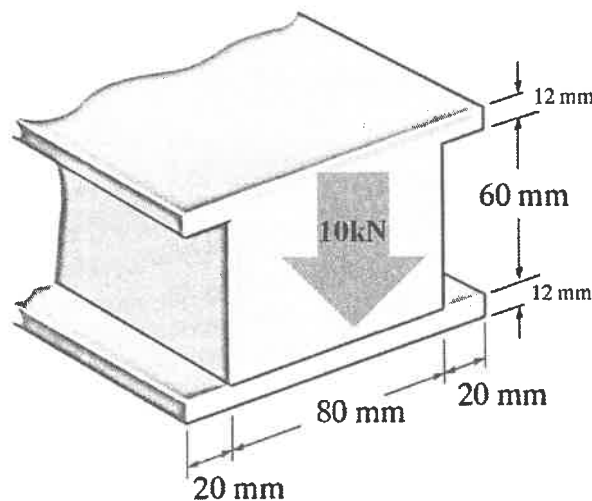


Fig. 4