



國立雲林科技大學

99 學年度碩士班暨碩士在職專班招生考試試題

系所：機械系

科目：工程數學(1)

1. Solve for the following differential equations.

(a)  $y' + y \tan x = \sin 2x$  (10%)

(b) 
$$\begin{cases} \frac{dx}{dt} + 2x + \frac{dy}{dt} + 6y = e^t \\ 2\frac{dx}{dt} + 3x + 3\frac{dy}{dt} + 8y = 2t \end{cases}$$
 (15%)

2. Solve for the following differential equation.

(a)  $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$  (15%)

$$u(0, t) = u(L, 0) = 0, \quad u(x, 0) = f(x)$$

(b) if  $f(x) = \begin{cases} x & \text{if } 0 < x < L/2 \\ L-x & \text{if } L/2 < x < L \end{cases}$  (10%)



Prob. 3 (30%)

$$\text{If } A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

Please find  $A^{26} + 2A^{15} + A$

Prob. 4 (20%)

Three vectors in  $\mathbb{R}^4$  are given as below.

$$\vec{e}_1 = \begin{bmatrix} 3 \\ -2 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{e}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix} \quad \text{and} \quad \vec{e}_3 = \begin{bmatrix} 3 \\ -1 \\ 7 \\ 0 \end{bmatrix}$$

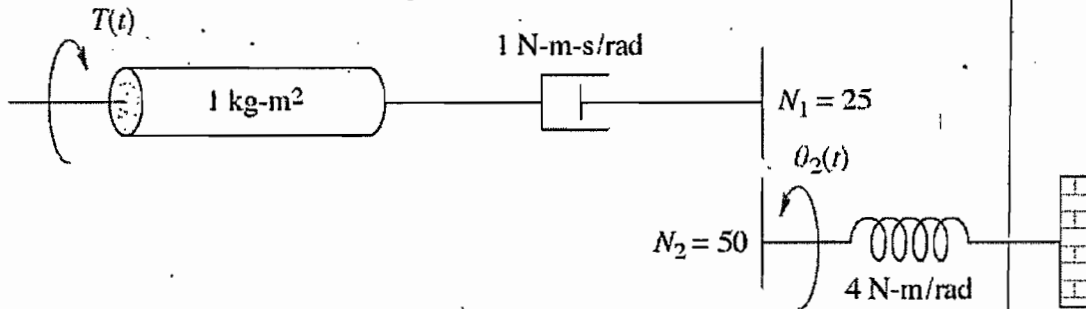
Are those vectors linearly independent?

If they are not, please find  $x$  and  $y$ , such that

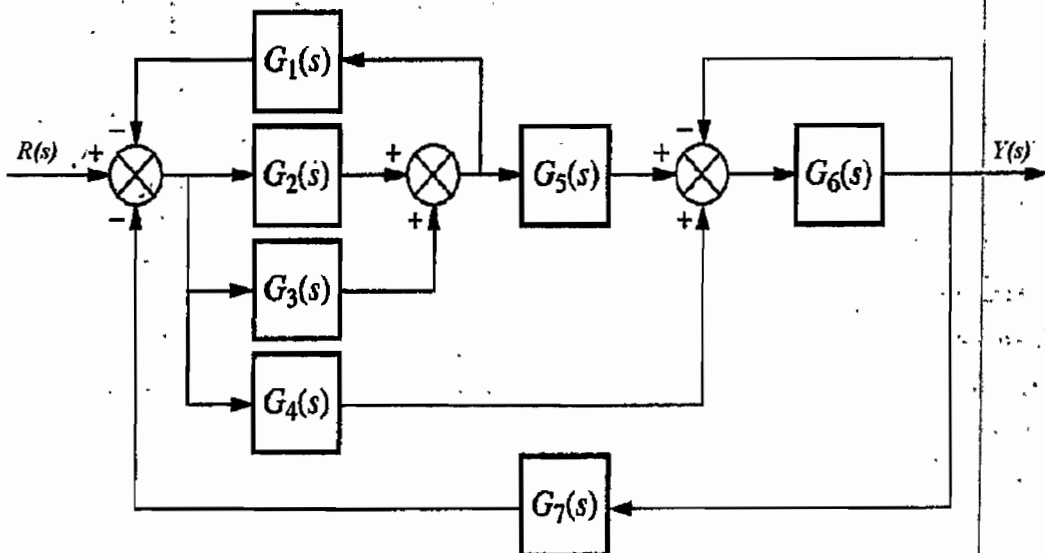
$$\vec{e}_3 = x\vec{e}_1 + y\vec{e}_2.$$



- (1) Find the transfer function  $G(s) = \theta_2(s) / T(s)$ , for the rotational mechanical system with gears shown below (25%) :

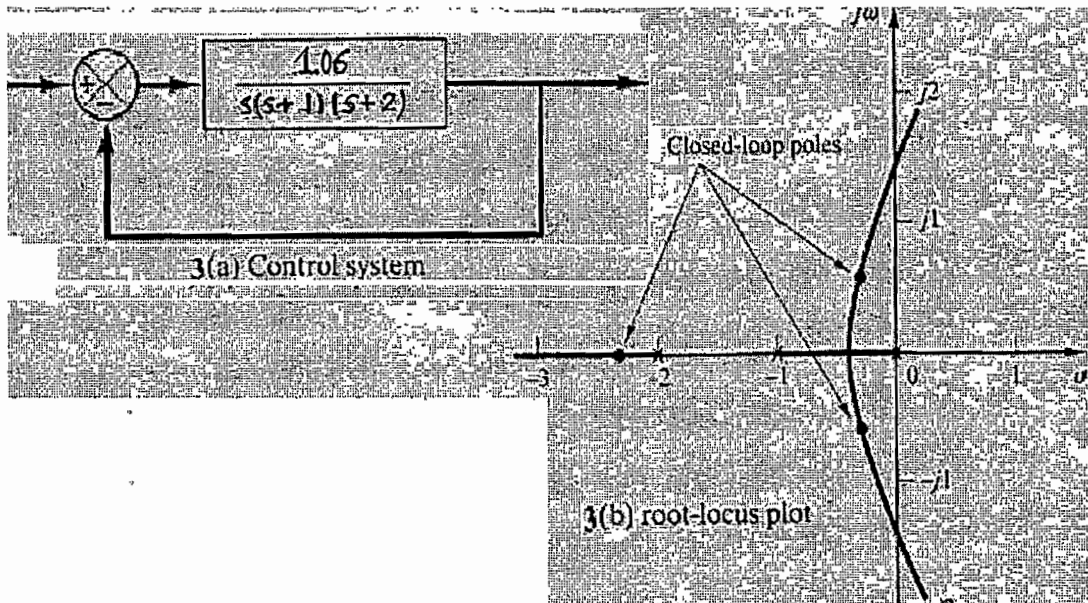


- (2) Reduce the block diagram shown below to a single block,  $T(s) = Y(s)/R(s)$ . (25%)

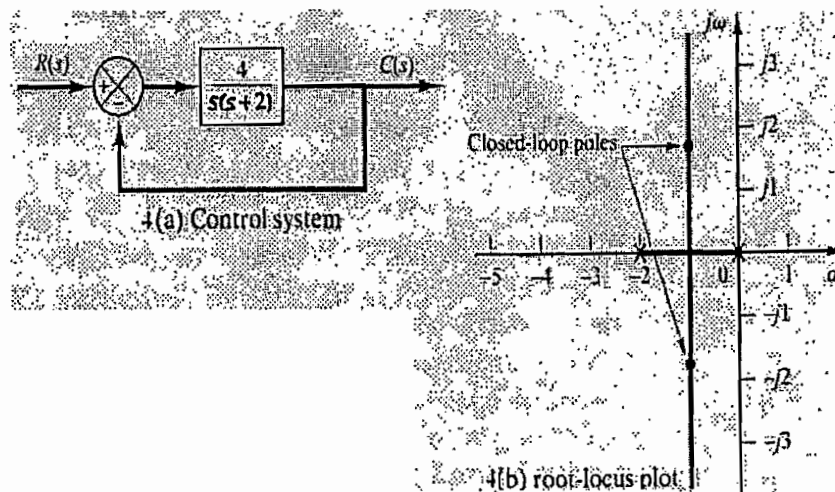




- (3) Consider the system shown in 3(a). The root-locus plot of the system is shown in 3(b). The dominant closed-loop poles are  $S = -0.3307 \pm j0.5864$ . The damping ratio of the dominant closed-loop poles is  $\zeta = 0.491$ . The undamped natural frequency of the dominant closed-loop poles is 0.673 rad/sec. It's desired to reduce the steady-state error of the system by a factor of 10, without changing the location of the dominant closed-loop poles. Please design a compensator to meet this specification. (25%)



- (4) Consider the system shown in 4(a). The root-locus plot of the system is shown in 4(b). The closed-loop poles are  $S = -1 \pm j\sqrt{3}$ . The damping ratio of the closed-loop poles is  $\zeta = 0.5$ . The undamped natural frequency of the closed-loop poles is 2 rad/sec. It's desired to modify the transient response of the system so that an undamped natural frequency  $\omega_n = 4$  rad/sec is obtained, without changing the damping ratio  $\zeta = 0.5$ . Please design a compensator to meet this specification. (25%)





1. Figure 1 shows the tensile engineering stress-strain behavior for a steel alloy.
- What is the modulus of elasticity?
  - What is the yield strength at a strain offset of 0.002?
  - What is the tensile strength?
  - Compute the modulus of resilience.
  - What is the ductility, in percent elongation?
- (20%)

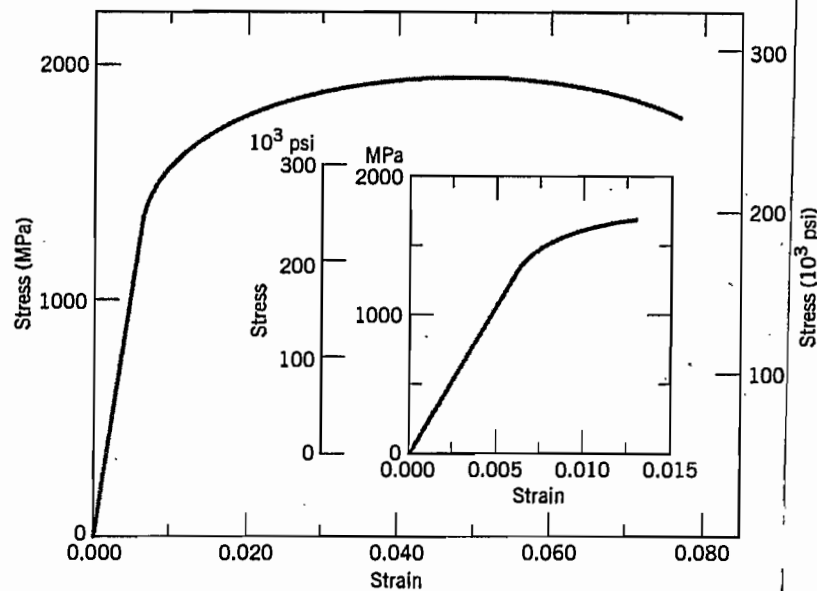


Fig. 1

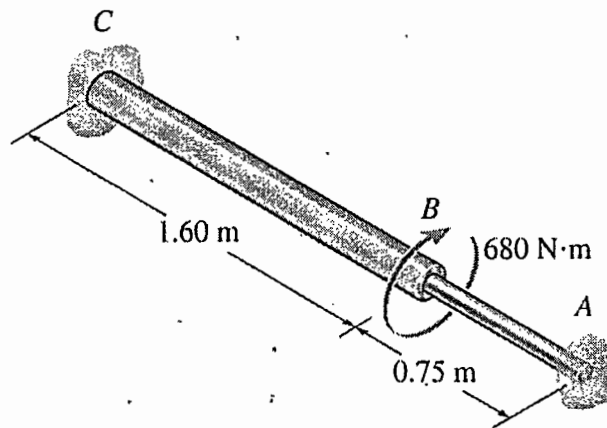
2. Cite advantages and disadvantages of hot working and cold working. (10%)
3. The three general classes of surface treatments are compositional modification, cold working and heat treating. Describe how each method can be used to increase the surface hardness of a steel component. (10%)
4. (a) Explain why the thermal conductivity of a single crystal specimen is slightly greater than a polycrystalline one of the same material.  
 (b) Explain why the thermal conductivity of a plain carbon steel is greater than for a stainless steel. (10%)



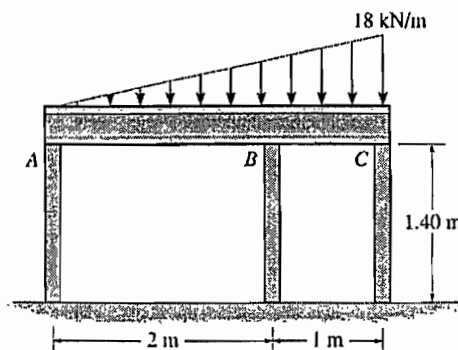
5. Please list and describe the principle of any two techniques of film deposition in the fabrication of microelectronics. (20%)
6. Please describe briefly the principle and the limitations of the following Non-destructive Testing Techniques. (20%)
- a. Liquid Penetrant Technique (滲探法)
  - b. Ultrasonic Inspection Technique (超音波檢測法)
  - c. Magnetic particle Inspection Technique (磁粒檢測法)
  - d. Radiographic Inspection Technique (射線檢測法)
7. Photoresist is one kind of polymers commonly used in the photolithography process for microelectronics. The most popular method to apply photoresist is spin-coating. Please list five factors that would affect the film thickness and uniformity of photoresist. (10%)



1. A rod is made from two segments:  $AB$  is steel and  $BC$  is brass. It is fixed at its ends and subjected to a torque of  $T=680 \text{ N}\cdot\text{m}$ . The steel portion has a diameter of 30 mm. The shear modulus of steel is  $G_{st} = 75 \text{ GPa}$ , and the shear modulus of brass is  $G_{br} = 39 \text{ GPa}$ .
- (a) Determine the required diameter of the brass portion so the reactions at the walls will be the same. (13%)
- (b) If the diameter of the brass portion is 50 mm, determine the absolute maximum shear stress in the shaft. (12%)



2. The horizontal beam is assumed to be rigid and supports the distributed load shown. Each support consists of a wooden post having a diameter of 120 mm and an unloaded (original) length of 1.40 m. The Young's modulus of wood is  $E_w = 12 \text{ GPa}$ .
- (a) Determine the vertical reactions at the supports. (18%)
- (b) Determine the angle of tilt of the beam after the load is applied. (7%)





3. A simple beam AB carrying two concentrated loads  $P = 5 \text{ kN}$  (Fig.3) has a rectangular cross section of width  $b = 100 \text{ mm}$  and height  $h = 100 \text{ mm}$ . The distance  $a$  from the end of the beam to one of the loads is  $0.5 \text{ m}$ . Determine the maximum normal and shear stresses in the beam. Disregard the weight of the beam itself. (20%)

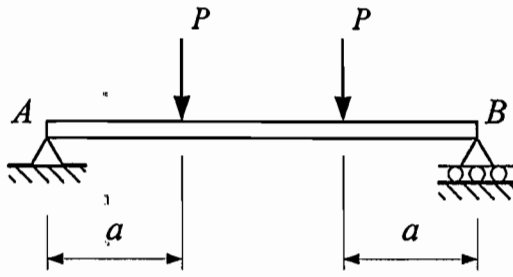


Fig.3

4. A simple beam with a uniformly distributed load  $w$  (Fig.4), (a) determine the reactions at support A and B, (b) draw the shear and moment diagrams for the beam, (c) calculate the maximum moment, (d) determine the equation of the deflection curve, (e) determine the maximum deflection at the middle of the beam. (30%)

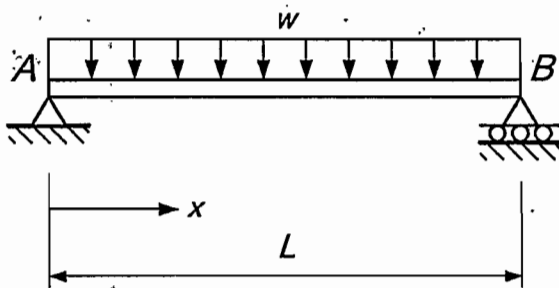
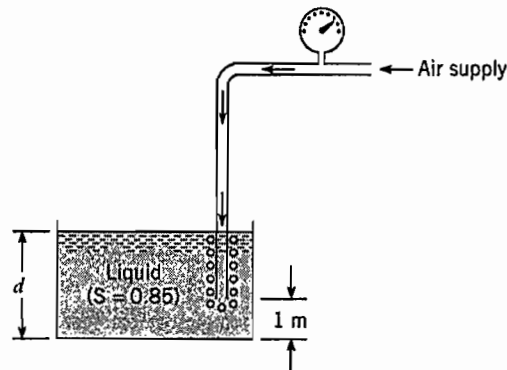


Fig.4

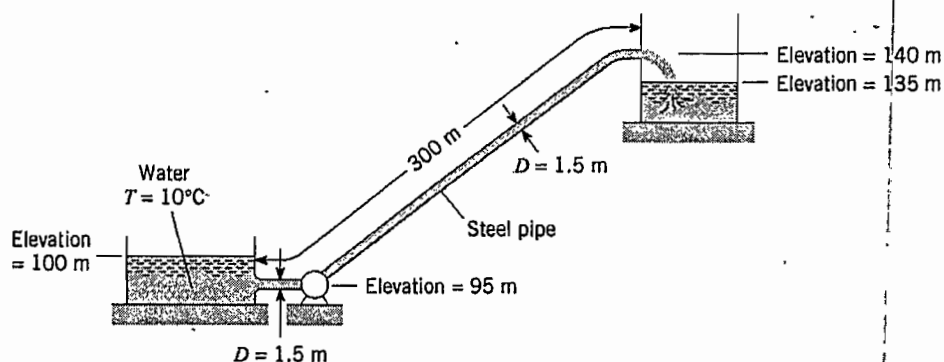




1. One means of determining the surface level of liquid in a tank is by discharging a small amount of air through a small tube, the end of which is submerged in the tank, and reading the pressure on the gage that is tapped into the tube. Then the level of the liquid surface in the tank can be calculated. If the pressure on the gage is 20kPa, what is the depth  $d$  of liquid in the tank? (the density of water is  $1000\text{kg/m}^3$ ;  $g = 9.81\text{m/s}^2$ )(25%)

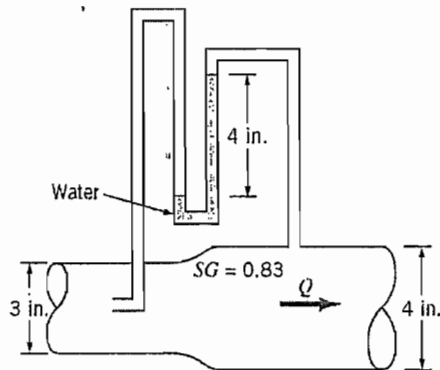


2. Water is pumped at a rate of  $20\text{m}^3/\text{s}$  from the reservoir and out through the pipe, which has a diameter of 1.5m. What power must be supplied to the water to effect this discharge? (25%)(assume friction factor  $f$  for this case is 0.010, the density of water is  $1000\text{kg/m}^3$ ,  $g = 9.81\text{m/s}^2$ , and the entrance loss coefficient  $K = 0.03$ )





3. Oil of specific gravity 0.83 flows in the pipe shown in Fig. If viscous effects are neglected, what is the flowrate? (gravity,  $g = 32.2 \text{ ft/s}^2$ ) (25%)



4. Two water jets of equal size and speed strike each other as shown in Fig. Determine the speed,  $V$  and the direction,  $\theta$  of the resulting combined jet. Gravity is negligible. (25%)

