



1. A 20 kg body ( $w$ ) is initially stationary on  $30^\circ$  incline as shown in Fig. 1. The coefficient of dynamic friction  $\mu_d$  between the block and incline is 0.3. What distance along the incline must the weight slide, if its speed is 12 m/s? (hint:  $g = 9.81 \text{ m/s}^2$ ). (25%)

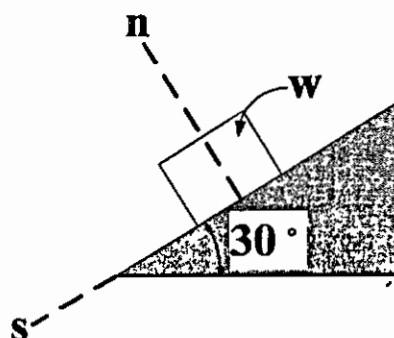


Fig. 1

2. (a) Derive the equation for coefficient of restitution  $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$ . [Fig. (推導)]

2(a) ~ 2(e)]. (10%)

- (b) The bag  $A$ , having a weight of 6 lb, is released from rest at the position  $\theta = 0^\circ$ , as shown in Fig. 2(f). After falling  $\theta = 90^\circ$ , it strikes a box  $B$  with  $v_B = 10 \text{ ft/s}$ , and having weight of 18 lb. If the coefficient of restitution between the bag and box is  $e = 0.5$ , determine the velocities of bag and box just after impact and the loss of energy during collision. (15%)

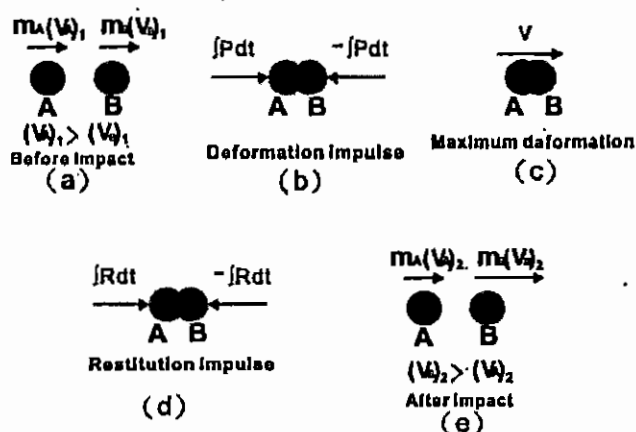
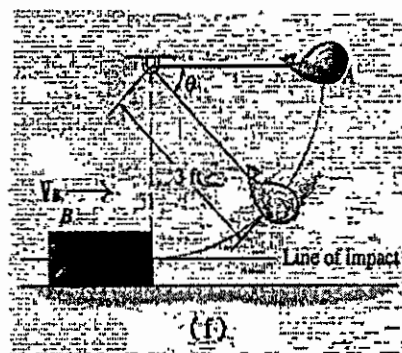


Fig. 2





3. In the engine system shown in Fig. 3, the 2 kg piston P is connected to the crank AB by the connecting rod BD of mass 1 kg. During a test of the system, crank AB is made to rotate with a constant angular velocity of 600 rpm clockwise. When the angle  $\theta=0^\circ$ , find
- The velocity and acceleration of the piston. (12%)
  - The forces exerted on the connecting rod at B and D. (Neglect the effect of the weight of the rod) (13%)

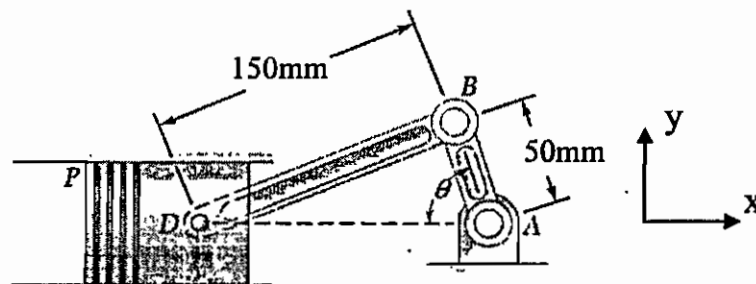


Fig. 3

4. Fig. 4 shows a planetary gear system. The radius of the sun gear (link 1) is 120 mm and that of the planetary gear (link 4) is 40 mm. Knowing that the sun gear has a constant angular velocity of 200 rpm clockwise and the ring gear (link 3) is stationary. Please determine
- The angular velocities of the planetary gear and the carrier (link 2). (12%)
  - The acceleration of the tooth of the planetary gear that is in contact with the ring gear. (13%)

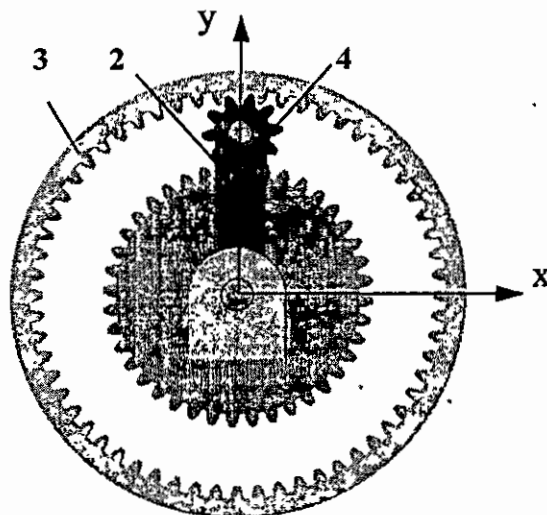


Fig. 4

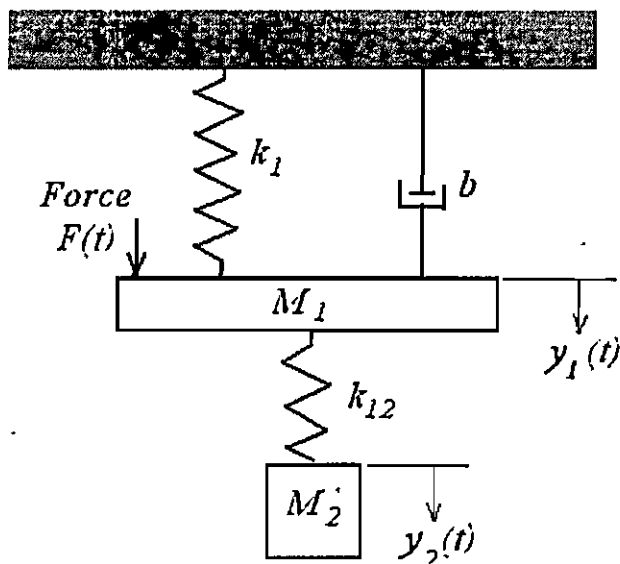


1. For the mechanical system shown below.

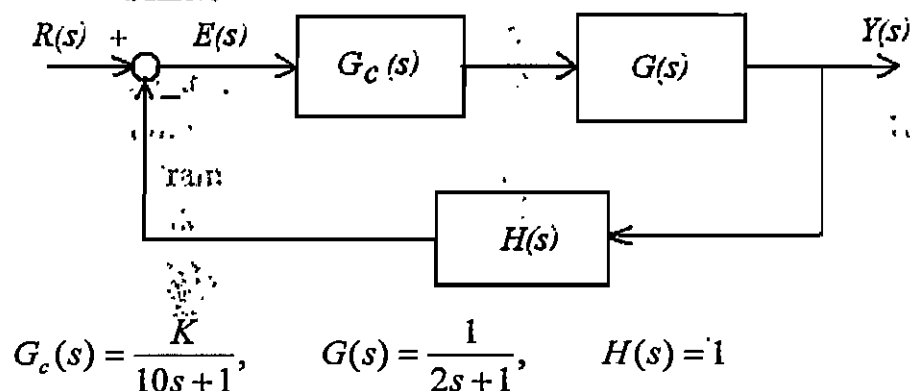
(a) Obtain the differential equations describing the system. (10%)

(b) Using Cramer's rule to solve the transfer function

$$G(s) = Y_1(s)/F(s). \text{ (15\%)}$$



2. For the block diagram shown below:



(a) Determine the closed-loop transfer function  $T(s) = Y(s)/R(s)$  (5%).

(b) Calculate the sensitivity  $S_K^T$  (10%).

(c) Determine the steady-state error for a unit step change  $R(s) = 1/s$  (5%).

(d) Calculate the value of  $K$  required for an allowable steady-state error of 2%. (5%)



3. Lag compensator design(25%)

Compensate the system of Figure 3.1, whose root locus is shown in Figure 3.2, to improve the steady state error by a factor of 10 if the system is operating with a damping ratio of 0.174.

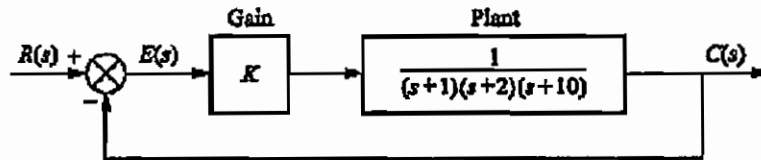


Figure 3.1

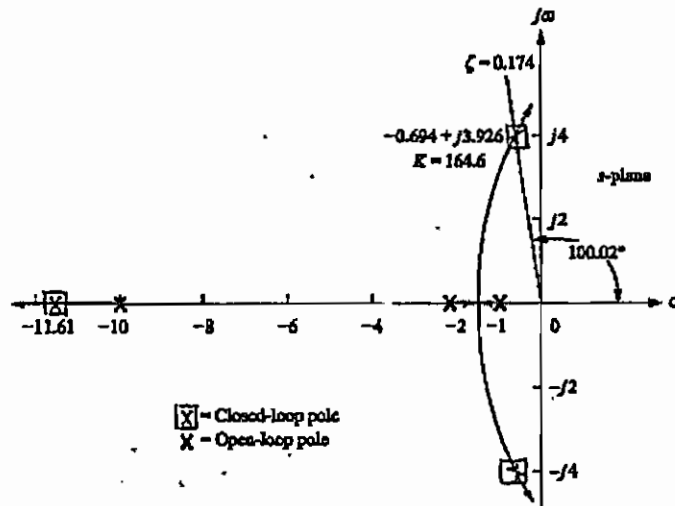


Figure 3.2

4. Lead compensator design(25%)

Compensate the system of Figure 4.1, whose root locus is shown in Figure 4.2, to reduce the settling time by a factor of 2 while maintaining 16% overshoot.

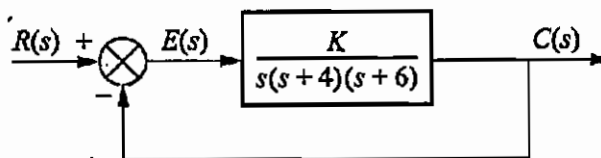


Figure 4.1

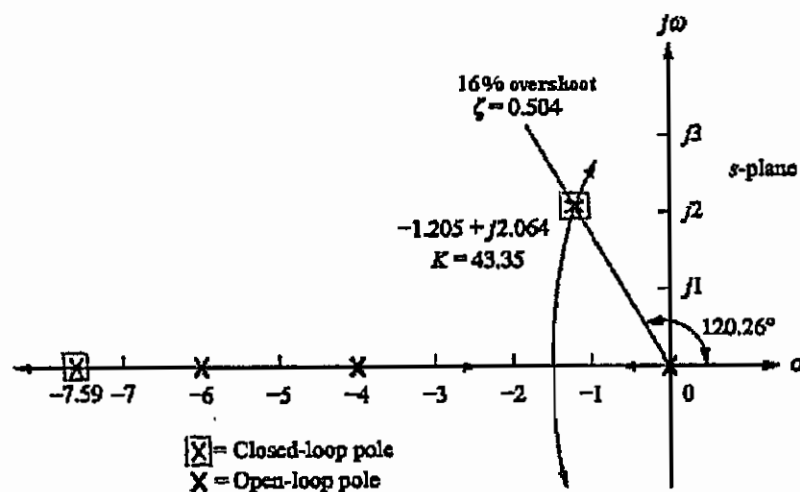


Figure 4.2



1. Solve the following differential equations.

(a)  $\cos x(e^{2y} - y) \frac{dy}{dx} = e^y \sin 2x, \quad y(0) = 0 \quad (10\%)$

(b)  $y''' - y'' + 100y' - 100y = 0, \quad y(0) = 4, \quad y'(0) = 11, \quad y''(0) = -299 \quad (15\%)$

2. Solve the following differential equation.

(a)  $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < L, \quad t > 0 \quad (15\%)$

$$\frac{\partial u}{\partial x}(0, t) = \frac{\partial u}{\partial x}(L, t) = 0 \quad \text{for } t > 0$$

$$u(x, 0) = f(x) \quad \text{for } 0 < x < L,$$

(b) if  $f(x) = \begin{cases} A & \text{for } 0 \leq x \leq \frac{L}{2} \\ 0 & \text{for } \frac{L}{2} \leq x \leq L \end{cases}$ , what will the solution be? (10%)



3. (30%)

Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation

given by reflecting across the plane  $-x_1 + x_2 + x_3 = 0$

(a) Show the matrix  $M$  representing  $T$  with respect to standard basis

$$\text{is: } M = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \quad (10\%)$$

(b) Find eigen values and eigen vectors of the matrix  $M$ . (10%)

(c) Find the inverse matrix  $M^{-1}$ . (10%)

4. (20%)

Find all the vectors in  $\mathbb{R}^3$

which are orthogonal to the vector  $[1, -1, 1]^T$  in  $\mathbb{R}^3$



1. The rigid bar shown in Fig. 1 is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250mm when no load is applied to the bar, and the temperature is  $T_1=20^\circ\text{C}$ . Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to  $T_2=80^\circ\text{C}$ . ( $G_{st}=200\text{GPa}$ ,  $\alpha_{st}=12\times 10^{-6}/^\circ\text{C}$ ,  $G_{Al}=73.1\text{GPa}$ ,  $\alpha_{Al}=23\times 10^{-6}/^\circ\text{C}$ ) (25%)

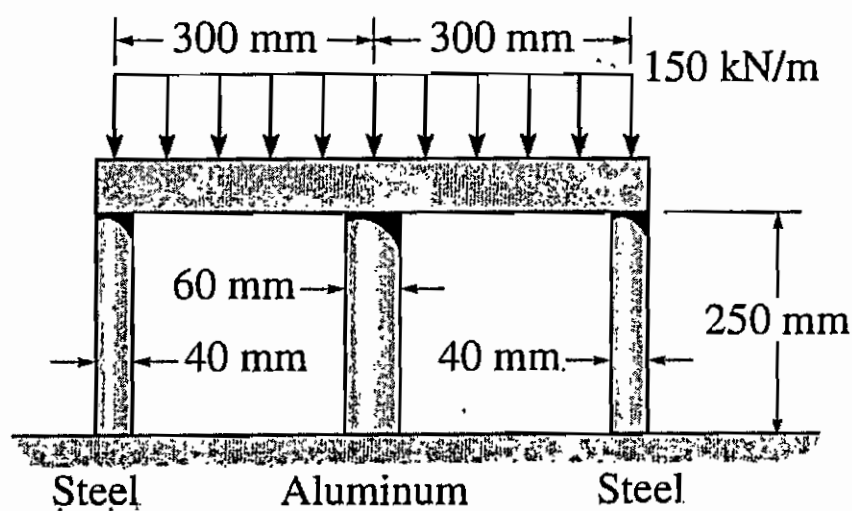


Fig.1

2. The wood beam is subjected to a load of 12kN. Determine the principal stresses at point A. (25%)

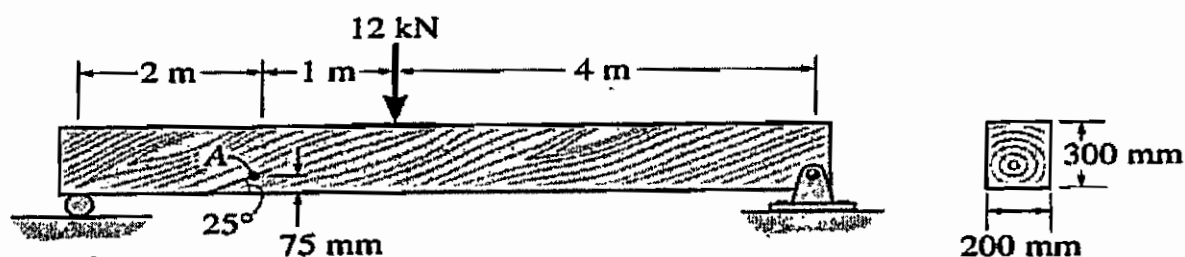


Fig.2



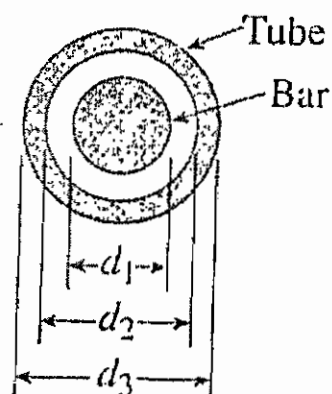
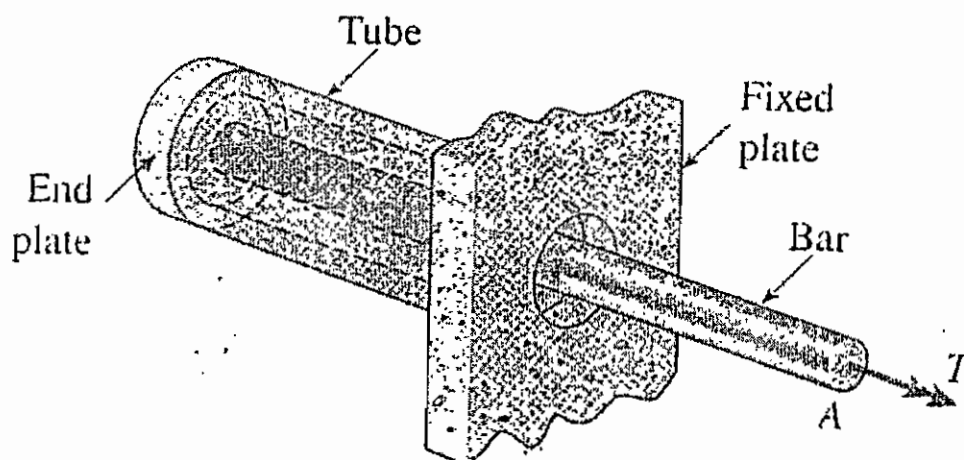
3. 雷射加工的特徵有那些，試列舉 10 點（10%）？  
對於銅及鋁等高反射率的材料進行加工時，必需注意那些事項才能獲得較佳的結果，試列舉 5 點（10%）？
4. 試說明 A F M（Abrasive Flow Machining）的原理（10%）？  
並說明使用 A F M 時，必需注意的事項（5%）？
5. 試說明電子束拋光的原理（10%）？  
並說明使用電子束拋光時，必需注意的事項（5%）？





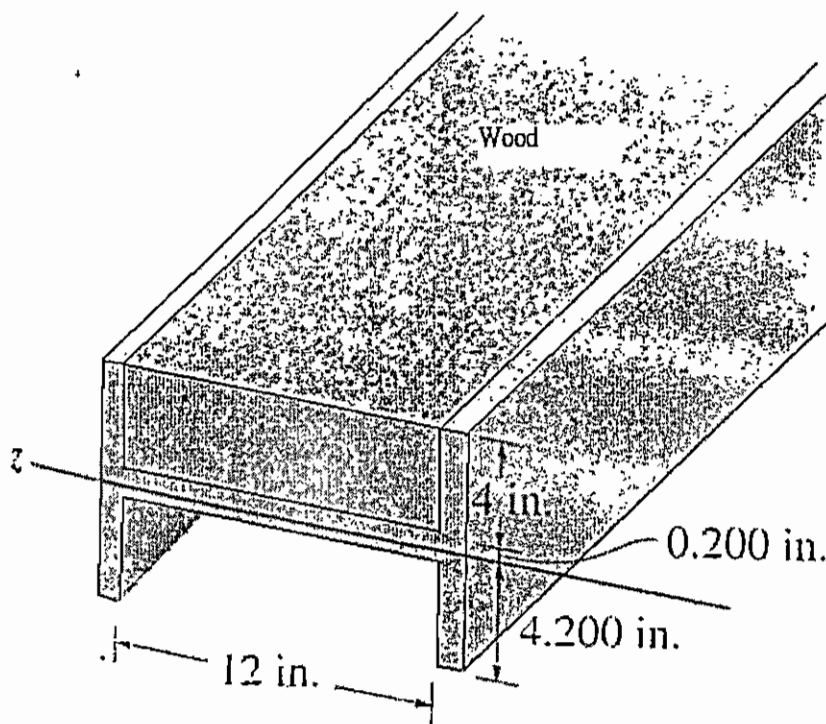
1. A circular tube of outer diameter  $d_3 = 70$  mm and inner diameter  $d_2 = 60$  mm is welded at the right-hand end to a fixed plate and at the left-hand end to a rigid end plate (see figure). A solid circular bar of diameter  $d_1 = 40$  mm is inside of, and concentric with, the tube. The bar passes through a hole in the fixed plate and is welded to the right end plate. The bar is 1.0 m long and the tube is half as long as the bar. A torque  $T = 1000$  N·m acts at end  $A$  of the bar. Also, both the bar and tube are made of an aluminum alloy with shear modulus of elasticity  $G = 27$  GPa.

- (a) Determine the maximum shear stresses in both the bar and tube. (15%)  
(b) Determine the angle of twist (in degrees) at end  $A$  of the bar. (10%)





2. In order to reinforce the steel beam, an oak board is placed between its flanges as shown. If the allowable normal stress for the steel is  $(\sigma_{allow})_{st} = 24 \text{ ksi}$ , and for the wood  $(\sigma_{allow})_w = 3 \text{ ksi}$ , determine the maximum bending moment the beam can support, with and without the wood reinforcement.  $E_{st} = 29 \times 10^3 \text{ ksi}$ ,  $E_w = 1.60 \times 10^3 \text{ ksi}$ . The moment of inertia of the steel beam is  $I_z = 20.3 \text{ in}^4$ , and its cross-sectional area is  $A = 8.79 \text{ in}^2$ . (25%)





3. Make a drawing for the element at A of the beam in Fig. 3 with horizontal and vertical sides, and show the stresses acting on it. Construct the corresponding Mohr circle. Determine the values of the principal stresses and the maximum shear stress. (25%)

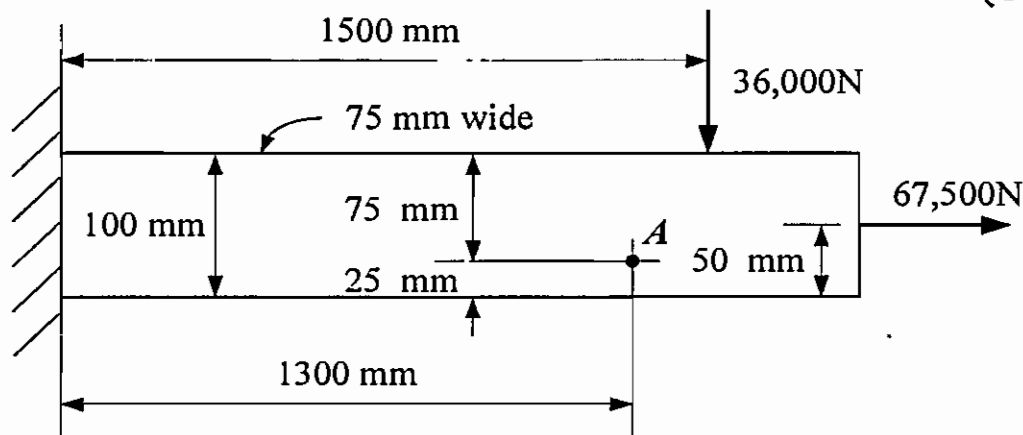


Fig. 3

4. Determine (a) the reaction forces at support A and B, (b) the beam's moment diagram, (c) the equation of the elastic curve for the beam using the x coordinate that is valid for  $0 \leq x \leq \frac{L}{2}$ , (d) the slope at A and beam's maximum deflection. (6%)  
(6%)  
(7%)  
EI is constant.

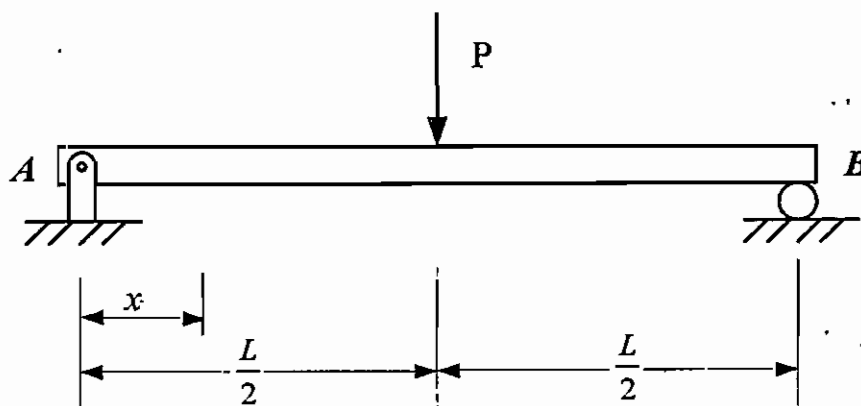


Fig. 4



1. The hydrometer shown in Fig.1 sinks 5.3cm in water ( $15^{\circ}\text{C}$ ). The bulb displaces  $1.0\text{cm}^3$ , and the stem area is  $0.1\text{cm}^2$ . a) Find the weight of the hydrometer. (10%)  
b) The top scale of the stem represents the lowest specific gravity it can measure or the highest? Why? (10%) c) If now the hydrometer is placed in a brine solution and sink 3.3cm ( $z=3.3\text{cm}$ ), then what is the specific gravity of that brine solution? (5%) (assume  $\rho_{\text{water}} = 1000\text{kg/m}^3$ ,  $g = 9.8\text{m/s}^2$ )

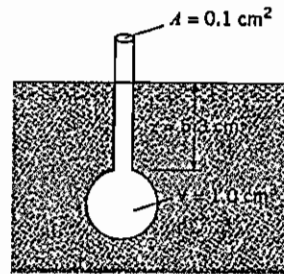


Fig.1

2. Fig.2 is the T-s diagram of water,  
a) What is the state of water in region I, II, III and at point 1, 3 respectively? (10%)  
b) What is point 2 called? (2%) What is the temperature at point 4 called? If humid air is cooled at constant pressure ( $P_2$ ), then, what will happen as the temperature drops below  $T_4$ ? (5%)  
c) Compare the magnitude of pressure  $P_1$  and  $P_2$ . (4%)  
d) Besides pressure and temperature, what extra information is needed in order to determine the properties of water in region II? (4%)

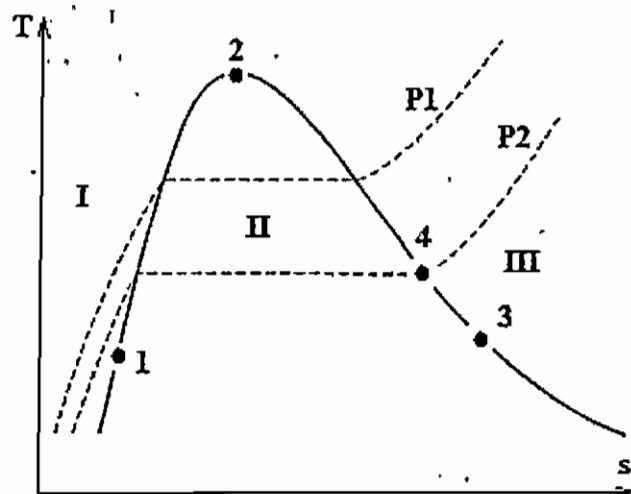
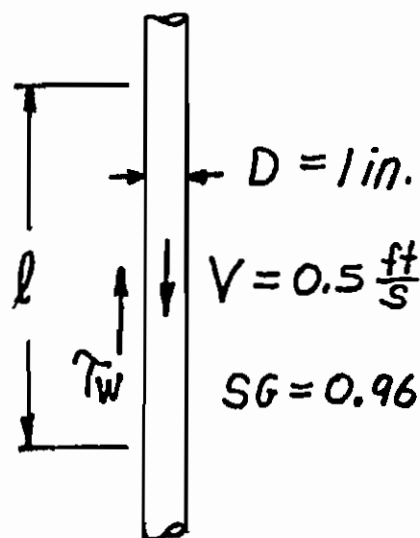


Fig.2



3. A fluid of specific gravity 0.96 flows steadily in a long vertical 1-in-diameter pipe with an average velocity of 0.50 ft/sec. If the pressure is constant throughout the fluid, what is the viscosity of the fluid? Determine the shear stress on the pipe wall. (25%)



4. The equation of state of a gas is given as  $\bar{v}(P + 10/\bar{v}^2) = R_u T$ , where the units of  $\bar{v}$  and  $P$  are  $\text{m}^3/\text{kmol}$  and  $\text{kPa}$ , respectively. Now 0.5 kmol of this gas is expanded in a quasi-equilibrium manner from 2 to 4  $\text{m}^3$  at a constant temperature of 300 K. Determine (a) the unit of the quantity 10 in the equation and (b) the work done during this isothermal expansion process. (25%, a: 13%, b: 12%)