



1. Solve the following differential equations:

(a) $ydx - 2xdy = 0$ (5%)

(b) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 3x$ (10%)

(c) $x\frac{dy}{dx} + 2y = xy^3$ (10%)

2. If matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix}$

(a) Find eigenvalues of A (5%)

(b) Find eigenvectors of A (10%)

(c) Find the diagonalized form of A (10%)

3. Let the matrix $A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$.

(a) (5%) Find the eigenvalues of A.

(b) (5%) Find the eigenvectors of A.

(c) (5%) Find the inverse of A.

(d) (5%) Find a matrix P such that $P^{-1}AP$ is a diagonal matrix.



4. Consider the function $f(x, y) = 2x^2y^3 + 6xy$.

(a) (5%) Find the gradient of $f(x, y)$ at the point $(1, 1)$.

(b) (5%) Find the directional derivative of $f(x, y)$ at the point $(1, 1)$ in the direction of a unit vector whose angle with the positive x -axis is $\pi/3$.

(c) (5%) Find an equation of the tangent plane to the graph of $z = f(x, y)$ at the point $(1, 1, 8)$.

5. (15%) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = 4 \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < \pi, \quad t > 0$$

subject to $u(0, t) = 0, \quad u(\pi, t) = 0, \quad u(x, 0) = 0, \quad \left. \frac{\partial u}{\partial t} \right|_{t=0} = \sin 2x - \sin 3x.$



本試題共四題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分

1. (a) Find $\frac{dy}{dx}$ of $e^{x^2y} = x + y$. [7%]
 (b) Find the limit of $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$. [8%]
 (c) Find $\partial z / \partial s$ for $z = e^r \cos \theta$, $r = st$, $\theta = \sqrt{s^2 + t^2}$. [10%]
2. (a) Find a power series representation for the function and determine the interval of convergence. $f(x) = \frac{1}{1 + 9x^2}$ [10%]
 (b) Find the point on the plane $x - y + z = 4$ that is closest to the point $(1, 2, 3)$. [15%]
3. Please find the integrations as below
 (i) $\int \frac{e^x}{e^x - 4} dx$ (10%)
 (ii) $\int \frac{x^2}{(x+2)^3} dx$ (15%)
4. Please find the integration as below
 (i) $\int x e^{x^2} dx$ (10%)
 (ii) $\int x e^{2x} dx$ (15%)



1. A particle moves along the hyperbolic spiral $r\theta = b$ shown in Fig. 1
 - (a) Determine the magnitude of the velocity of the particle in terms of b , angle θ , and angular velocity ω . (12%)
 - (b) Knowing that ω is constant, determine the magnitude of the acceleration of the particle in terms of b , angle θ , and angular velocity ω . (13%)

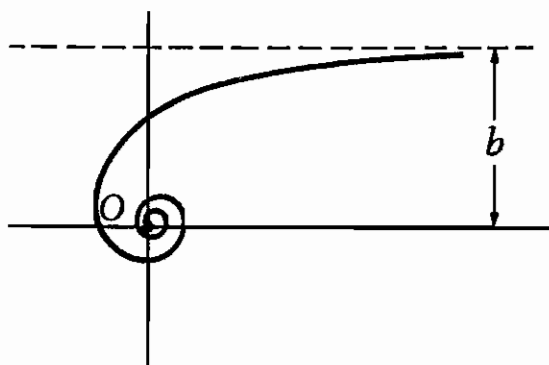


Fig. 1

2. Two rotating rods are connected by slider block P. The rod attached at A rotates with a constant angular velocity ω_A and angular acceleration α_A for the position angle θ_A shown in Fig. 2.
 - (a) Derive the velocity equation to show the expression for the velocity of slider block P and the angular velocity of the rod attached at B. (12%)
 - (b) Derive the acceleration equation to show the expression for the acceleration of slider block P. (13%)

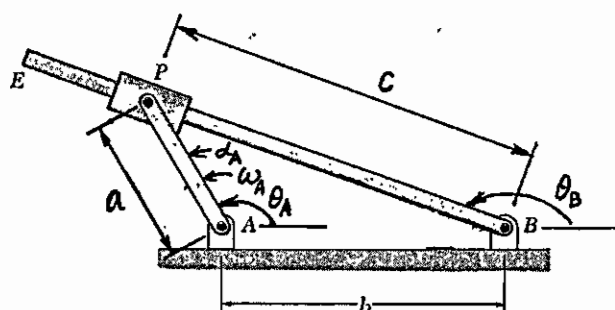


Fig. 2



3. The uniform pole has a mass of 20 kg and falls from rest when $\theta = 90^\circ$ until it strikes the edge at A , $\theta = 60^\circ$. If the pole then begins to pivot about this point A after contact, determine the pole's angular velocity just after the impact. Assume that the pole does not slip at B as it falls until it strikes A . Please solve the problem based on "conservation of energy" and "conservation of angular momentum." See Fig. 3 below. (25%)

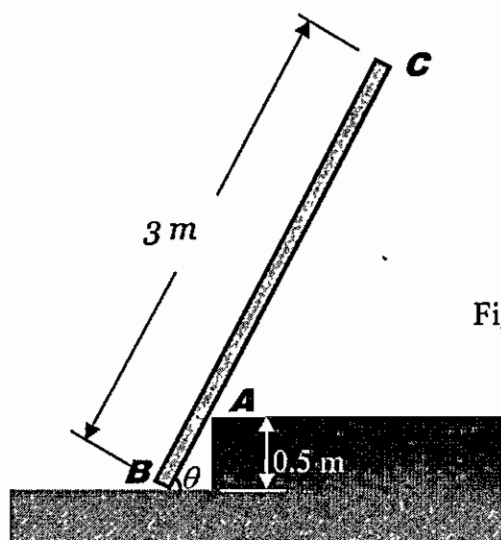


Fig. 3

4. At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 180 mm/s^2 , both measured relative to the disk and directed away from O . If at the same instant the disk has the angular velocity and an acceleration shown, determine the velocity and acceleration of the ball at the instant. See Fig. 4 below. (25%)

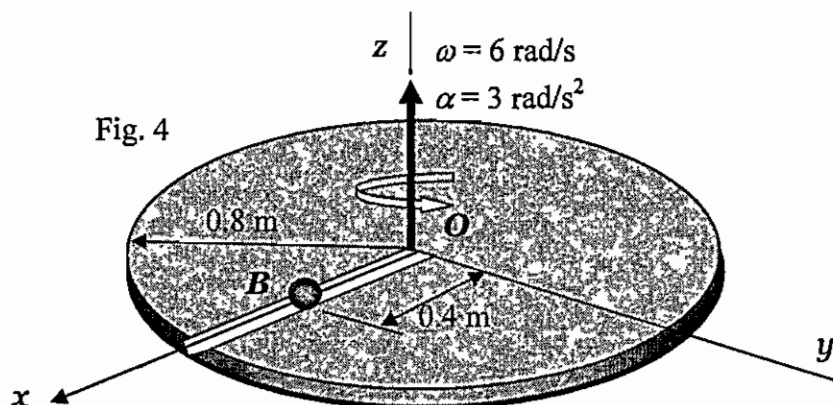


Fig. 4



1. (1) 請說明粉末冶金之優點及缺點。(15%)
(2) 簡述粉末冶金之基本製程。(10%)

2. 請簡述電鍍的基本原理(10%)，並說明兩種可以改善電鍍均厚能力的方法。(15%)

3. 請舉出最少五種與機械製造有關的主要磨耗類型，並敘述之(25%)

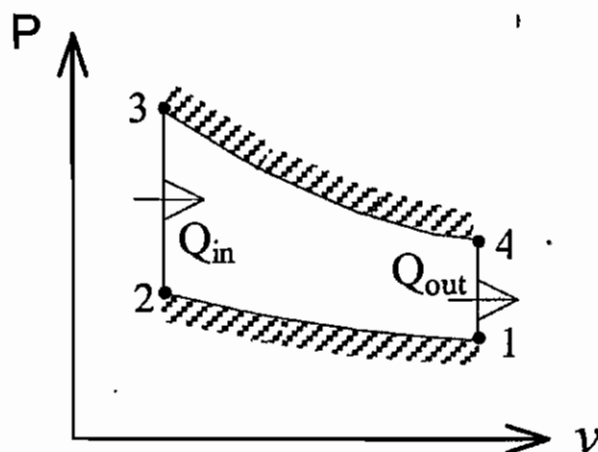
4. 若欲加工一圓形加工物，請舉出三種不同製造方式，並依加工物本身、各加工法適用性及優缺點敘述之(25%)



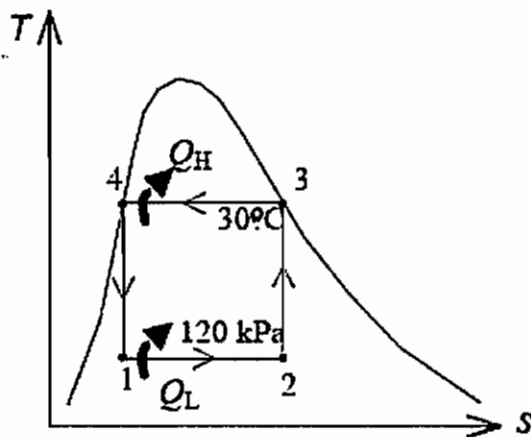
1. A Carnot heat engine receives heat from a reservoir at 900°C at a rate of 700 kJ/min and rejects the waste heat to the ambient air at 27°C . The entire work output of the heat engine is used to drive a Carnot refrigerator that removes heat from the refrigerated space at -5°C and transfer it to the same ambient air at 27°C . Determine (a) the power output from the Carnot heat engine to the refrigerator (b) the heat removal from the refrigerated space by the Carnot refrigerator (c) the total rate of heat rejection to the ambient by the Carnot heat engine and refrigerator. (25%)
2. R-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine (a) the quality at the outlet of the capillary tube, (b) the temperature drop across the capillary tube. (c) If air enters the same capillary tube at 0.8 MPa and 27°C and outlet pressure from the capillary is 0.101 MPa. Assuming air is a perfect gas for the above conditions, air gas constant, $R = 0.287 \text{ kJ/kg-K}$, please determine the air density at the outlet of the capillary tube. (25%)



3. The compression ratio of an air-standard Otto cycle is 9.5. Prior to the isentropic process, the air is at 100 kPa, 17 °C and 600 cm³. The temperature at the end of the isentropic expansion process is 800 K. Using the specific heat values at the room temperature, determine (a) the highest temperature and pressure in the cycle; (b) the amount of heat transferred in, in KJ; (c) the thermal efficiency; and (d) the mean effective pressure. ($C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$, $C_v = 0.718 \text{ kJ/kg}\cdot\text{K}$, and $k = 1.4$; $R = 0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K}$) (25%)



4. A steady-flow Carnot refrigeration cycle uses refrigerant 134a as the working fluid. The refrigerant changes from saturated vapor to saturated liquid at 30 °C in the condenser as it rejects heat. The evaporator pressure is 120 kPa. Determine (a) the coefficient of performance, (b) the amount of heat absorbed from the refrigerated space, and (c) the net work input. ($T_{\text{sat}} @ 120 \text{ kPa} = -22.36 \text{ °C}$) (25%)





Saturated refrigerant-134a—Temperature table

Temp., T °C	Press., P _{sat} MPa	Specific volume, m ³ /kg		Internal energy, kJ/kg		Enthalpy, kJ/kg			Entropy, kJ/kg · K	
		Sat. liquid, v _f	Sat. vapor, v _g	Sat. liquid, u _f	Sat. vapor, u _g	Sat. liquid, h _f	Evap., h _{fg}	Sat. vapor, h _g	Sat. liquid, s _f	Sat. vapor, s _g
-40	0.05164	0.0007055	0.3569	-0.04	204.45	0.00	222.88	222.88	0.0000	0.9560
-36	0.06332	0.0007113	0.2947	4.68	206.73	4.73	220.67	225.40	0.0201	0.9506
-32	0.07704	0.0007172	0.2451	9.47	209.01	9.52	218.37	227.90	0.0401	0.9456
-28	0.09305	0.0007233	0.2052	14.31	211.29	14.37	216.01	230.38	0.0600	0.9411
-26	0.10199	0.0007265	0.1882	16.75	212.43	16.82	214.80	231.62	0.0699	0.9390
-24	0.11160	0.0007296	0.1728	19.21	213.57	19.29	213.57	232.85	0.0798	0.9370
-22	0.12192	0.0007328	0.1590	21.68	214.70	21.77	212.32	234.08	0.0897	0.9351
-20	0.13299	0.0007361	0.1464	24.17	215.84	24.26	211.05	235.31	0.0996	0.9332
-18	0.14483	0.0007395	0.1350	26.67	216.97	26.77	209.76	236.53	0.1094	0.9315
-16	0.15748	0.0007428	0.1247	29.18	218.10	29.30	208.45	237.74	0.1192	0.9298
-12	0.18540	0.0007498	0.1068	34.25	220.36	34.39	205.77	240.15	0.1388	0.9267
-8	0.21704	0.0007569	0.0919	39.38	222.50	39.54	203.00	242.54	0.1583	0.9239
-4	0.25274	0.0007644	0.0794	44.56	224.84	44.75	200.15	244.90	0.1777	0.9213
0	0.29282	0.0007721	0.0689	49.79	227.06	50.02	197.21	247.23	0.1970	0.9190
4	0.33765	0.0007801	0.0600	55.08	229.27	55.35	194.19	249.53	0.2162	0.9169
8	0.38756	0.0007884	0.0525	60.43	231.46	60.73	191.07	251.80	0.2354	0.9150
12	0.44294	0.0007971	0.0460	65.83	233.63	66.18	187.85	254.03	0.2545	0.9132
16	0.50415	0.0008062	0.0405	71.29	235.78	71.69	184.52	256.22	0.2735	0.9116
20	0.57160	0.0008157	0.0358	76.80	237.91	77.26	181.09	258.35	0.2924	0.9102
24	0.64566	0.0008257	0.0317	82.37	240.01	82.90	177.55	260.45	0.3113	0.9089
26	0.68530	0.0008309	0.0298	85.18	241.05	85.75	175.73	261.48	0.3208	0.9082
28	0.72675	0.0008362	0.0281	88.00	242.08	88.61	173.89	262.50	0.3302	0.9076
30	0.77006	0.0008417	0.0255	90.84	243.10	91.49	172.00	263.50	0.3396	0.9070
32	0.81528	0.0008473	0.0250	93.70	244.12	94.39	170.09	264.48	0.3490	0.9064
34	0.86247	0.0008530	0.0236	96.58	245.12	97.31	168.14	265.45	0.3584	0.9058
36	0.91168	0.0008590	0.0223	99.47	246.11	100.25	166.15	266.40	0.3678	0.9053
38	0.96298	0.0008651	0.0210	102.38	247.09	103.21	164.12	267.33	0.3772	0.9047
40	1.0164	0.0008714	0.0199	105.30	248.06	106.19	162.05	268.24	0.3866	0.9041
42	1.0720	0.0008780	0.0188	108.25	249.02	109.19	159.94	269.14	0.3960	0.9035
44	1.1299	0.0008847	0.0177	111.22	249.96	112.22	157.79	270.01	0.4054	0.9030
48	1.2526	0.0008989	0.0159	117.22	251.79	118.35	153.33	271.68	0.4243	0.9017
52	1.3851	0.0009142	0.0142	123.31	253.55	124.58	148.66	273.24	0.4432	0.9004
56	1.5278	0.0009308	0.0127	129.51	255.23	130.93	143.75	274.68	0.4622	0.8990
60	1.6813	0.0009488	0.0114	135.82	256.81	137.42	138.57	275.99	0.4814	0.8973
70	2.1162	0.0010027	0.0086	152.22	260.15	154.34	124.08	278.43	0.5302	0.8918
80	2.6324	0.0010766	0.0064	169.88	262.14	172.71	106.41	279.12	0.5814	0.8827
90	3.2435	0.0011949	0.0046	189.82	261.34	193.69	82.63	276.32	0.6380	0.8655
100	3.9742	0.0015443	0.0027	218.60	248.49	224.74	34.40	259.13	0.7196	0.8117

R-134a

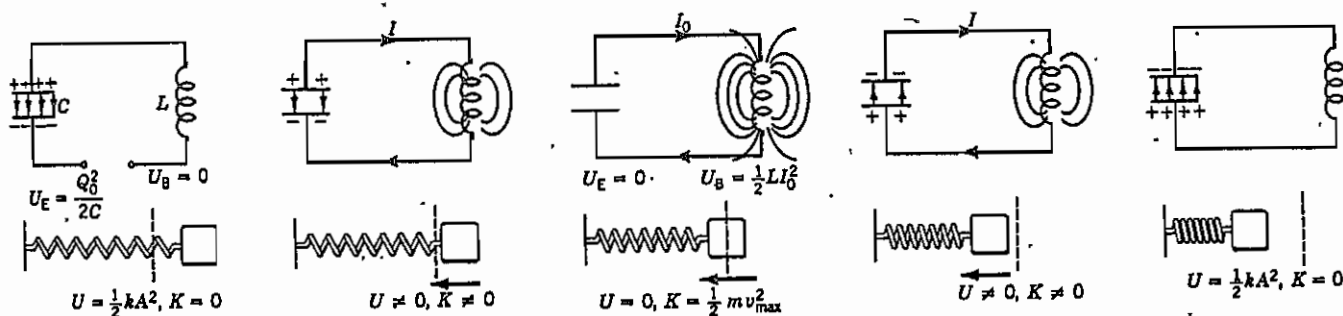


1. 試繪圖說明放電加工的原理? (5%)並說明玻璃、 Al_2O_3 陶瓷等電氣絕緣性的材料，是否可以利用放電加工法來進行加工?如果不可以→請說明理由(5%)
如果可以→請說明理由，並繪圖提出可以加工的方法(15%)。
注意→沒有繪圖不計分。
2. 一高速鋼螺旋齒平銑刀有 10 齒做平面銑削，每刀每齒每迴轉的進刀量為 0.45mm，若主軸每分鐘 100 轉，其總進刀量為何?(25%)



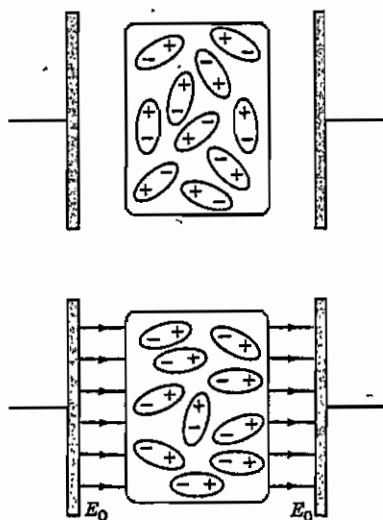
3. “LC 電路振盪系統”與“彈簧-質量振盪系統”之說明如下圖。
- 請說明下述兩振盪系統間的類似性。(15%)
 - 設兩振盪系統均無能量損耗，試說明其一個週期的運動行為。(10%)

LC 電路振盪系統



彈簧-質量振盪系統

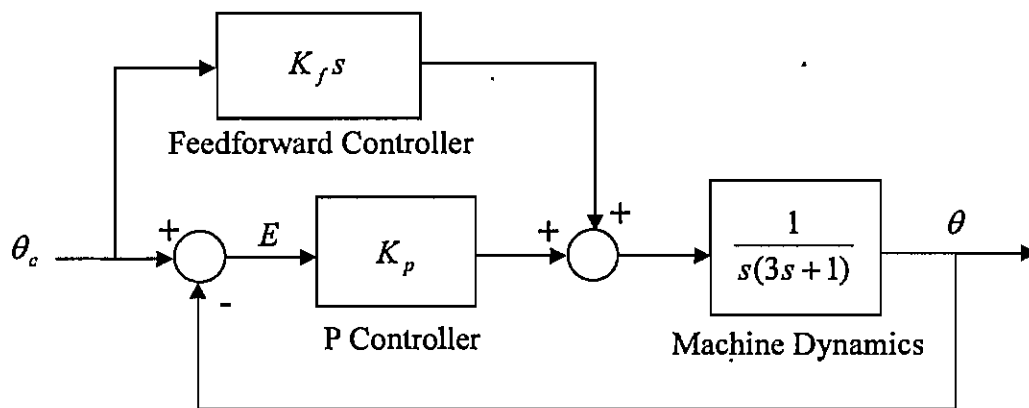
4. 兩平行電容板未充電時，兩電容板間的絕緣物質未被極化、顯現不帶電現象，兩平行電容板充電後，兩電容板間的絕緣物質被極化、顯現帶電現象；如下圖所示。
- 試說明，為何兩平行電容板充電後，兩電容板間的絕緣物質會顯現帶電現象。(10%)
 - 試利用下圖解釋，若兩平行電容板無限制地持續充電，兩電容板間的絕緣物質會被打穿，而失去絕緣性。(15%)





本試題共四題，共計 100 分，請依題號作答並將答案寫在答案卷上，違者不予計分。

1. The model of a servo drive system is shown below, where θ_c is the position command, θ is the position feedback, and E is the position error.



- (1) Find the transfer function between position command and position error, i.e. $E(s)/\theta_c(s)$.

(10%)

- (2) If $K_f = 1$ and the position command is a “ramped input”, find the steady state position error.

(5%)

- (3) Plot the root locus for $0 \leq K_p \leq 3$, and show critical K_p values which determine the shape of the root locus.

(10%)



2. For the loudspeaker system shown below, the electrical circuit generates a current i which is proportional to the force applied on the mass M :

$$f = k \cdot i \quad \dots\dots\dots(2.1)$$

and the mechanical system has a damping factor D . The differential equation representing the electrical dynamics of the system is known to be:

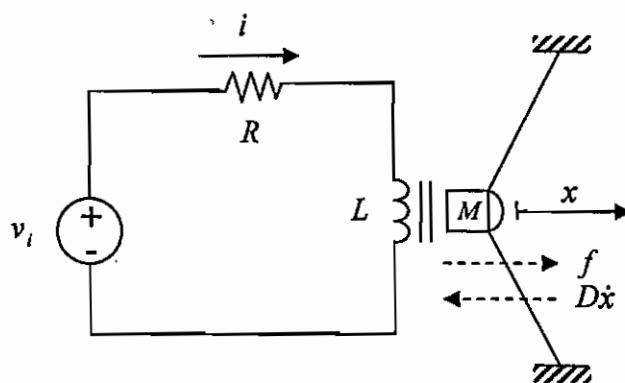
$$L \frac{di}{dt} + R \cdot i = v_i - k \frac{dx}{dt} \quad \dots\dots\dots(2.2)$$

- (1) Taking the Laplace Transform of (2.2), represent the coil current $I(s)$ using voltage input $V_i(s)$ and mass displacement $X(s)$.

(10%)

- (2) Using (2.1), (2.2) and mechanical dynamics, derive the transfer function $X(s)/V_i(s)$.

(15%)





3. A feedback control system has a characteristic equation:

$$s^3 + (1 + K)s^2 + 10s + (5 + 15K) = 0$$

The parameter K must be positive. Please find:

- (a) The maximal value of K before the system becomes unstable. (10%)
- (b) The frequency of oscillation when K is equal to the maximal value. (10%)

4. A feedback control system has a loop transfer function:

$$G(s)H(s) = \frac{100}{s\left(\frac{s}{10} + 1\right)\left(\frac{s}{100} + 1\right)}$$

- (a) Determine the corner (break) frequencies for the Bode plot. (5%)
- (b) Sketch the Bode plots. (15%)
- (c) Estimate the Gain Margin and Phase Margin. (10%)



1. The members in Fig. 1 have a neat fit at the time of assembly. Find the force caused by an increase in temperature of 50°C . Supports are immovable.

$$\alpha_{\text{steel}} = 11.7 \times 10^{-6} \text{ mm/mm}^{\circ}\text{C}, E_{\text{steel}} = 206,900 \text{ MPa}$$

$$\alpha_{\text{brass}} = 18.4 \times 10^{-6} \text{ mm/mm}^{\circ}\text{C}, E_{\text{brass}} = 103,400 \text{ MPa}$$

α : coefficient of thermal expansion, E : modulus of elasticity

(25%)

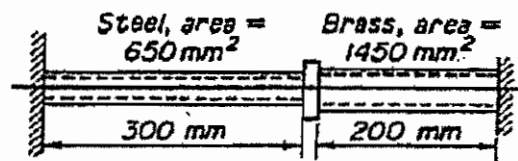


Fig. 1

2. Find the reactions and also the value of the bending stress at a point 1.5 m from the left end for the beam of Fig. 2.

(25%)

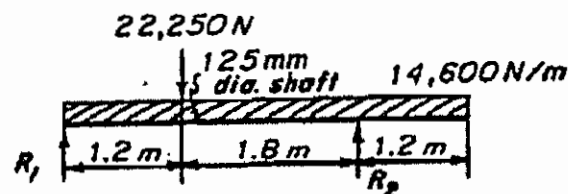


Fig. 2



3. When the torque T is applied to the bar, it produces a state of stress as shown. (a) Determine the maximum in-plane shear stress and their directions, 並求在此面上所作用的 normal stress 大小。 (b) Determine the principal stress and their directions. (c) Draw the Mohr's circle for this case. (d) 若材料因為受到此一負荷而斷裂，說明當材料是鑄鐵時，材料將如何斷裂（請畫出來），並解釋其原因。 (25%)

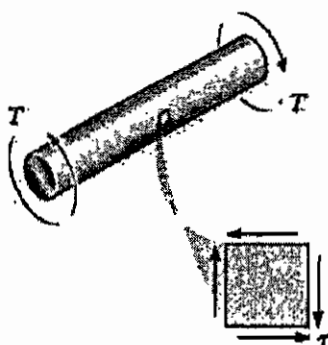


Figure for Problem 3

4. A cantilever beam AB is subjected to a triangularly distributed load of maximum intensity w_0 as shown. The beam has constant moment of inertia (I) and Young's modulus (E), and has length L . (a) Determine the reactions (both shear force V and moment M) at the support A . (b) Draw the shear diagram (V - x curve) and the moment diagram (M - x curve) for $0 \leq x \leq L$. (c) Determine the equation of the elastic curve for the beam. (d) Determine the angle of rotation θ_B and the deflection δ_B at the free end B . (25%)

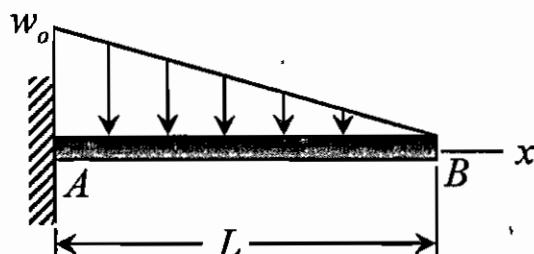
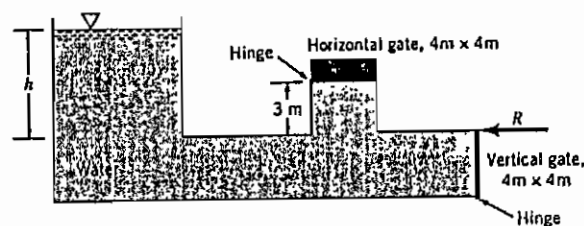


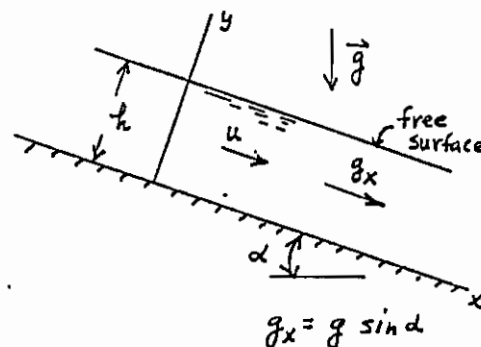
Figure for Problem 4



1. Two square gates close two openings in a conduit connected to an open tank of water as shown in the following figure. When the water depth, h , reaches 5 meter it is desired that both gates open at the same time. Determine the weight of the homogeneous horizontal gate and the horizontal force, R , acting on the vertical gate that is required to keep the gates closed until this depth is reached. The weight of the vertical gate is negligible, and both gates are hinged at one end as shown. Friction in the hinge is negligible. 25%

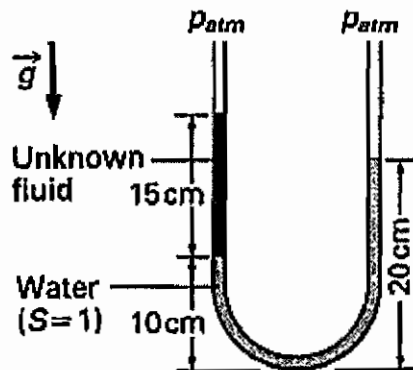


2. A layer of viscous liquid of constant thickness (no velocity perpendicular to plate) flows steadily down an infinite, inclined plane. Determine, by means of the Navier-Stokes equations, the relationship between the thickness of the layer and the discharge per unit width. The flow is laminar, and assume air resistance is negligible so that the shearing stress at the free surface is zero. 25%





3. What is the specific gravity (S) of the unknown fluid in the left leg of the U-tube manometer shown in the figure? (25%)



4. The syringe showed in the figure has an inside diameter of 1.0 cm. The syringe is held vertically, and the plunger is pushed upward at a rate of 4.0 cm/s. Find the maximum height to which the water will rise if the outlet measures 2.0 mm in diameter. (25%) ($g = 9.81 \text{ m/s}^2$)

