



國立雲林科技大學

九十學年度研究所碩士班入學考試試題

系所：機械系

科目：工程數學

Prob. 1

The equation is given as below:

$$10y' + y = 1, \text{ with } y(0) = 1$$

- (1) Is it exact? (You must do the test before giving the result.) (5%)
- (2) If it is not, please find its integrating factor to make it exact. (5%)
- (3) Then, find its potential function, and solve $y(x)$ in explicit form. (10%)
- (4) Sketch the graph of $y(x)$. (5%)

Note: The integration constant must be determined from initial condition.

Prob. 2

Given a system of equations as below

$$\dot{x}_1 = x_2 \quad \text{with } x_1(0) = 0$$

$$\dot{x}_2 = x_3 \quad x_2(0) = 0$$

$$\dot{x}_3 = -x_1 - 3x_2 - 3x_3 \quad x_3(0) = 1$$

Solve the above system of equations for $x_1(t)$, $x_2(t)$ and $x_3(t)$. (25%)



3. For a simple closed curve C composed of the boundary of the region in the first quadrant that is bounded by the graphs of $y=x$ and $y=x^3$, as shown in Fig. Prob. 3:

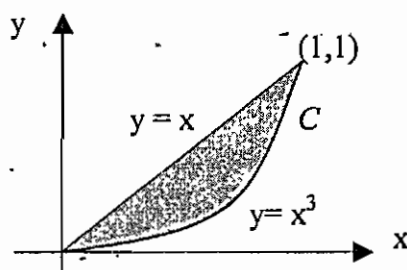


Fig. Prob. 3

(1) Evaluate the area integral on the region R bounded by curve C (the shaded area in Fig. Prob. 3):

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

, where $P=y^2$ and $Q=x$. [15%]

(2) For a force field $\mathbf{F} = y^2 \mathbf{i} + x \mathbf{j}$ acting along curve C , the work W done by \mathbf{F} is defined as:

$$W = \oint_C \mathbf{F} \cdot d\mathbf{r}$$

where $d\mathbf{r} = dx \mathbf{i} + dy \mathbf{j}$. Use the Green's theorem and the result of (1) to find W . [10%].

4.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

B.C.

$$\frac{\partial u}{\partial x}(x=0)=0, \quad \frac{\partial u}{\partial x}(x=a)=0, \quad \text{for } 0 < y < b$$

$$u(x, 0)=0, \quad u(x, b)=1, \quad \text{for } 0 < x < a$$

(1) Let $u(x, y) = X(x)Y(y)$, then show the new boundary conditions associated with X and Y . [10%]

(2) Solve u . [15%]



1. 試求 $\int \frac{x^3}{\sqrt{1+x^2}} dx = ?$ [10%]

2. 試求雙曲擺線 $x^{2/3} + y^{2/3} = a^{2/3}$ 之全長, 並繪圖說明。 [15%]

3. 設 S 為一區域由 $r = \cos 2\theta$, $\theta = -\pi/4$, $\theta = \pi/4$, 所圍成, 若密度函數為 $f(r, \theta) = r$, 試求其質心。 [15%]

4. 試繪出 $f(x) = x + 4/x$ 之圖形。 [10%]

5. What is the maximum directional derivative of the function f defined by

$$f(x, y, z) = x^2 + y^2 + z^2 \text{ at point } (a, b, c)? \quad (10\%)$$

Note: Point (a, b, c) means that the projections of the point on the x , y , and z axes have coordinates a , b , and c , respectively, in the rectangular coordinate system in space.

6. Find $\int_0^{\pi/2} x^2 \sin(x) dx$. (15%)

7. Show that

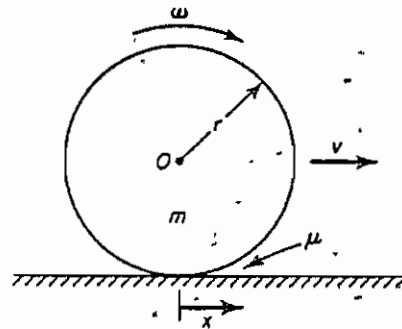
$$\int_x^1 \frac{dt}{1+t^2} = \int_1^{1/x} \frac{dt}{1+t^2} \quad \text{if } x > 0. \quad (15\%)$$

8. Find $\int_0^{\pi/2} \cos(2nx) \sin(x) dx$, where n is an integer. (10%)

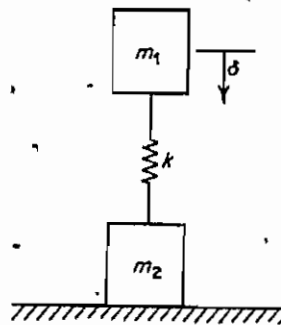


1. A uniform sphere of mass m and radius r moves with planar motion along a horizontal surface. Assuming a coefficient of sliding friction μ and initial conditions $\chi(0)=0$, $v(0)=v_0$, $w(0)=w_0$, where $v_0 > r w_0$ find: (a) the velocity of the sphere for large t ; (b) the displacement χ at which sliding stops. [25%]

註：七為時間

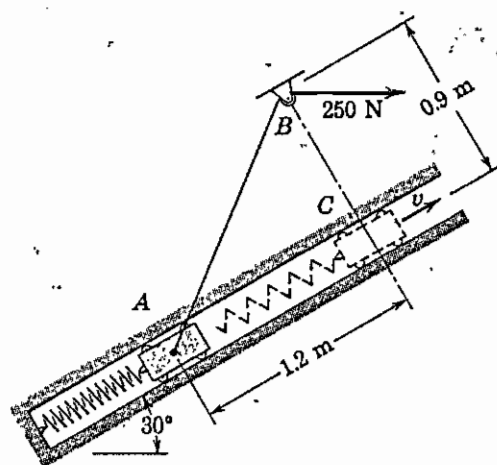


2. What is the minimum spring compression δ necessary to cause m_2 to leave the floor after m_1 is suddenly released with zero velocity? Measure δ from the unstressed length of the spring and assume that all motion is in the vertical direction. [25%]

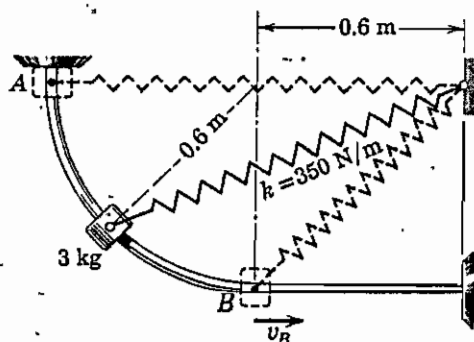




3. The 10-kg slider A moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position A, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v of the slider as it passes point C. [25%]



4. The 3-kg slider is released from rest at point A and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 350 N/m and has an unstretched length of 0.6 m. Determine the velocity of the slider as it passes position B. [25%]





1. 試說明固體力學與流體力學所探討之內容與方法有何異同？試描述純彈性固體受力作用所表現之行爲？又如果是純黏性流體呢？
(10%)
2. 圓管流常見之流量-壓力(Q-P)關係式爲 $Q = \pi P R^4 / 8 \eta L$ ，其中 R:管半徑， η : 黏度，L:管長。請問黏度單位爲何？以此公式說明心肌梗塞或高血壓如何造成？
(10%)
3. 就你專業所知，水力、風力、火力及太陽能發電之原理爲何？各有何優缺點？以及就台灣現有能源及環保環境，何者較具經濟及可行性？試詳述你的理由。
(15%)
4. 何謂應力與應變？單位爲何？一般量測材料彈性模數(elastic modulus, E)，多以拉伸試驗機量測應力-應變關係曲線而得。因此拉伸試驗機需具有哪些感測器或裝置。又需結合何種裝置，以求得材料浦松比(Poisson ratio)。
(15%)



5. 何謂材料彈性變形與塑性變形。(10%)
6. 致動器(Actuator)使用氣壓、液壓與電動此三種方式來作定位控制，有何差異。(10%)
7. 請說明定位準確度(accuracy)與重複定位精度(repeatability)，比較兩者相異處。(10%)
8. 熱傳遞時，請說明 Conduction、Convection、Radiation 之熱傳遞方式。(10%)
9. 機械製造方面，請比較 Arc welding 與 Spot welding 兩者相異之處。(10%)



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1. [25%] Air is compressed from 16 psia, 80 F to 100 psia, 400 F in a reciprocating piston compressor. Calculate the change in the specific entropy of the air caused by this compression. (the air c_v is 0.1711 Btu/lb-R and c_p is 0.2397 Btu/lb-R)
2. [25%] Some engineers have suggested that small motor vehicles can be propelled by expanding compressed air stored in an on-board storage tank through an air motor. What is the maximum amount of work that can be produced by such a system containing 2 kilograms of air at 1000 kPa, 327 C? ($R = 8.314$ kJ/kg-moleK, $c_v = 0.7165$ kJ/kg-K and $c_p = 1.0035$ kJ/kg-K)



3. (25%) Briefly answers the following questions:

- PCMs'(Phase Change Materials) are solid materials with different melting temperatures designed to recover waste heat based on the concept of latent heat. How are you going to arrange their orders(排列順序), concerning their different melting temperatures, relative to the flowing path of the flue gas? Why? (9%)
- Air under atmospheric pressure is going to be compressed to a much higher pressure. What kind of process will give the minimum work that is required to do the job? Isothermal, Adiabatic or Polytropic process? Why? (8%)
- The entropy of a hot baked potato decreases as it cools. Is this a violation of the increase of entropy principle? Explain. (8%)

4. (25%) A steam power plant operates on an ideal regenerative Rankine cycle. Steam enters the turbine at 6 MPa and 450°C and is condensed in the condenser at 20 kPa. Steam is extracted from the turbine at 0.4 MPa to heat the feedwater in an open feedwater heater. Water leaves the feedwater heater as a saturated liquid.

- Show the cycle on a T-s diagram. (15%), and
- Calculate the total work input to the cycle per kilogram of feedwater. (10%)

$$h_{f@20\text{kPa}} = 251.40\text{kJ/kg}, \quad v_{f@20\text{kPa}} = 0.001017\text{m}^3/\text{kg}$$

$$h_{f@0.4\text{MPa}} = 604.74\text{kJ/kg}, \quad v_{f@0.4\text{MPa}} = 0.001084\text{m}^3/\text{kg}$$

$$h_{g@6\text{MPa}, 450^\circ\text{C}} = 3301.8\text{kJ/kg}, \quad s_{g@6\text{MPa}, 450^\circ\text{C}} = 6.7193\text{kJ}/(\text{kg}\cdot\text{K})$$



1. (1) 假設你現在是某一工廠的工程師，欲生產製造某一產品，試說明你會考慮到哪些配合的因素（例如：加工方法、材料等）以完成之。請完整而詳細地說明你的想法，以充分展現出你思考的深度與廣度。你可以舉出幾個大項，再逐項說明每一大項包括哪些相關的細節。 [15 %]
- (2) 說明在一個製造模具的機械工場中，應該有哪些措施與設備，以提高作業人員的安全性。 [10 %]
2. 針對金屬材料之：[a] toughness; [b] ductility; [c] forgeability; [d] machinability; [e] weldability，請逐一解釋，並各別說明這些性質在機械製造上：(1) 為何很重要？ (2) 可用何種方式表示之？ (3) 量測之方法？ [25 %]



3. 不銹鋼管管內徑 1mm，長 20 mm，壁厚 0.1mm。可能的加工製造方式有哪些？少量與大量有何不同考量？又如果內壁要求表面粗糙度 $Ra=0.05\mu m$ 以下，有哪些可能之表面處理方式？ (25%)
4. 市售手機外殼多以塑膠射出或金屬壓鑄成型製作，請問兩者在機台、模具與成型條件有何異同？塑膠與金屬於模內流動行為有何異同及可能衍生問題有哪些？冷卻固化過程有何異同及可能衍生問題有哪些？如何解決？ (25%)



1. Consider the system shown in Fig. 1.

- (A) If $r = 1$ and $k_1 = 4$, determine the range of k_2 for which the closed-loop system is stable. (2%)
- (B) If $r = 1$ and $k_1 = 4$, find the value of k_2 for which the output y keeps on oscillations at a constant amplitude. Also determine the frequency of oscillations. (10%)
- (C) If $r = \sin 10t$, where t denotes the time in seconds, determine the ranges of k_1 and k_2 for which the output y will oscillate at a constant amplitude and at a frequency of 10 rad/sec in the steady state. (5%)
- (D) If $r = \sin 10t$, and $k_1 = k_2 = 5$, what is the amplitude of oscillations in the steady state? (8%)

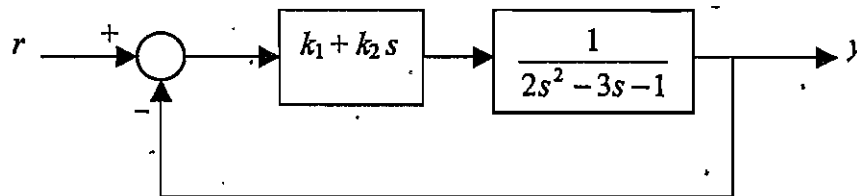


Fig. 1

2. Consider the system described by

$$\frac{Y(s)}{U(s)} = \frac{400}{(s + 200)(s + 2)}.$$

(A) Sketch the unit step response. Estimate the time it takes for the output to reach 90% of its final (i.e. steady state) value. (10%)

(B) What is the bandwidth of the system? (5%)

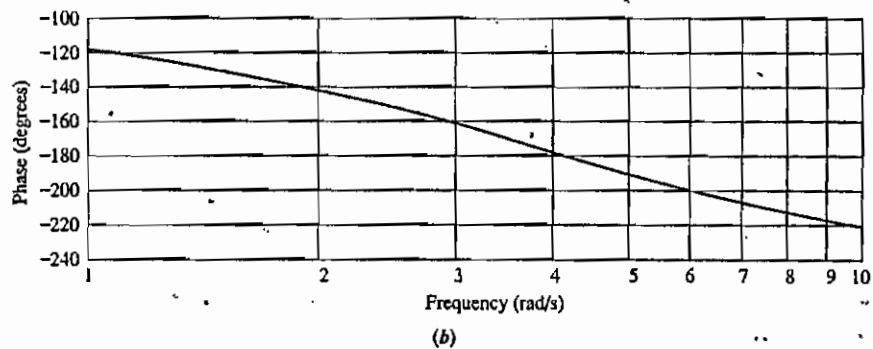
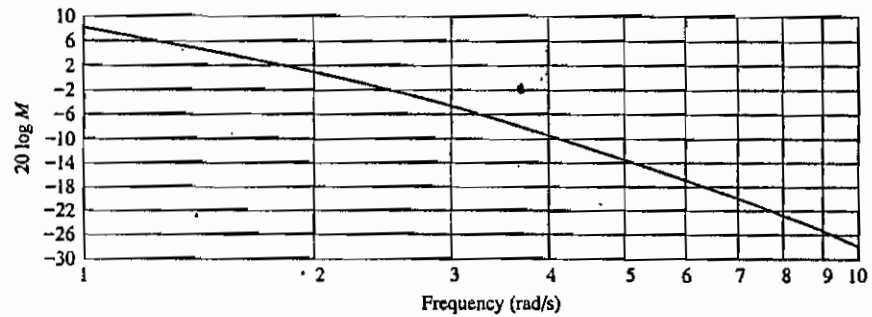
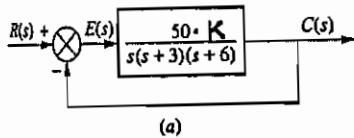
3. (10%) Sketch the impulse response of the system described by

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3 + s^2 + 4s}.$$



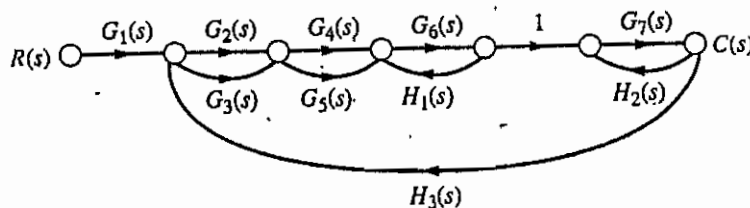
4. A closed-loop control system with its Bode diagram (for $K=1$) is shown in the following Fig.(a) and Fig.(b) respectively.

- Find the gain margin and phase margin graphically. (5%)
- Sketch the possible Nyquist diagram in GH-plane using the gain margin and phase margin. (Hint: Exact calculation and drawing are not necessary, sketch only) (10%)
- Use the Bode diagram to determine the range of K for the feedback system to be stable. (10%)



5. For the signal-flow-graph shown below:

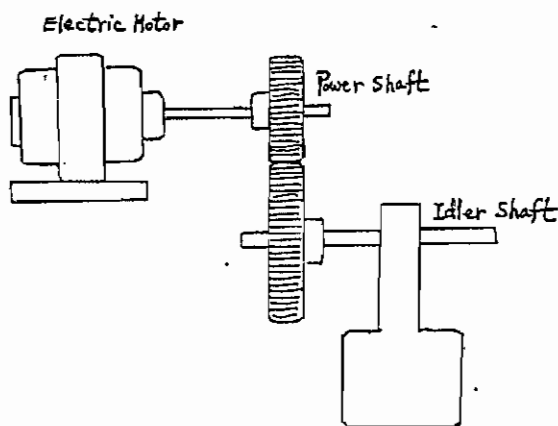
- Find the transfer function $T(s)=C(s)/R(s)$. (15%)
- Transform and represent this signal-flow-graph into the form of a block diagram. (10%)





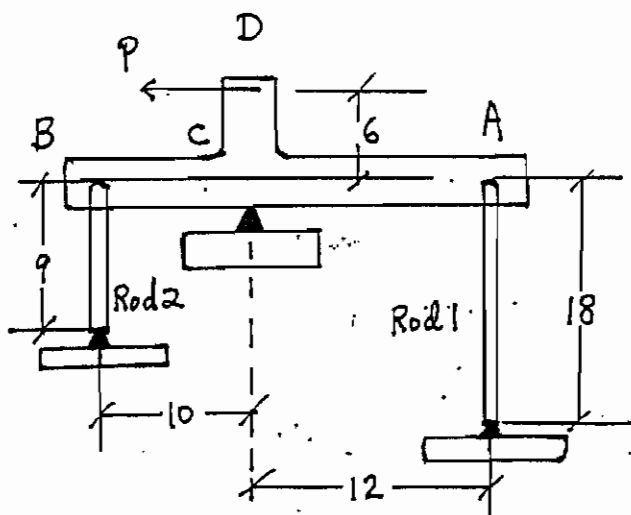
1. An electric motor delivers 50 *hp* through a power shaft with diameter 2.0 *inches* at a rotational speed 500 *rpm* to an idler shaft with diameter 3.0 *inches* as shown below. The gear of the power shaft has 16 teeth and the teeth number of the gear of the idler shaft is 96. Find out:

- The torque in *lb-ft* generated by the power shaft, 5%
- The shear stress in *psi* in the power shaft, 5%
- The output power in *hp* from the idler shaft, 5%
- The rotational speed in *rpm* of the idler shaft, 5%
- The shear stress in *psi* in the idler shaft. 5%



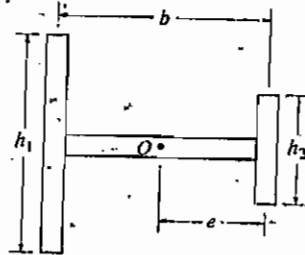
2. A pin-connected rigid member is loaded and supported as shown below. Rods 1 and 2 are made of A36 structural steel bar with an elastic modulus of $E = 29,000 \text{ ksi}$. The cross-sectional area of rod 1 is 0.11 in^2 and its length is 18 *inches*. Rod 2 has a cross-sectional area of 0.2 in^2 and a length of 9 *inches*. A load is applied at point D of the rigid member. The dimensions of the structure are given in *inches*. The allowable normal stress in the steel bars is 24 *ksi*.

- Compute the allowable load in *kips* that may be applied at point D. 5%
- Which bar should be assumed to have the allowable normal stress? 5%
- Compute the normal stress in *ksi* in the bar that is not assumed to have the allowable normal stress. 5%
- Compute the deflection in *inches* at the point of load application D. 5%
- Which direction does the tip of the rigid member move to? 5%

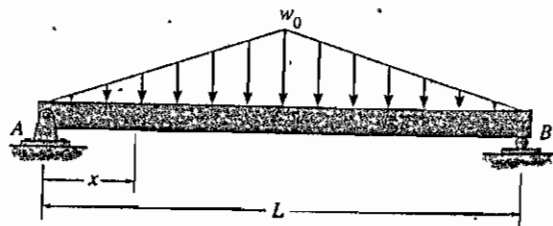




3. Determine the location e , of the shear center, point O , for the thin-walled member having the cross section shown. The member segments have the same thickness t . (25%)



4. Consider the simply supported beam loaded as shown below. EI is constant.
- Determine the elastic curve for the beam using the x coordinate $0 \leq x \leq L/2$. (13%)
 - Determine the slope at A . (6%)
 - Determine the maximum deflection of the beam. (6%)





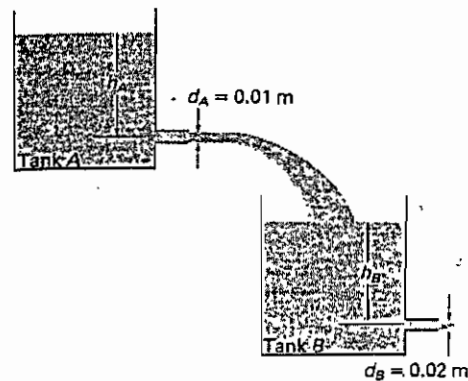
1. A long cylinder rod of 100-mm radius consists of a nuclear reacting material generating $24,000 \text{ W/m}^3$ uniformly throughout its volume. The conductivity of the nuclear reacting material is $k = 0.5 \text{ W/(m K)}$. The nuclear rod is tightly covered (encapsulated) by a tube cylinder having an outer radius of 200-mm and a thermal conductivity of 4 W/(m K) . There is no contact resistance between the nuclear rod and the tube cylinder. The outer surface of the tube cylinder is surrounded by a fluid at 373 K , and the convection coefficient between the fluid and the outer surface is $20 \text{ W/(m}^2 \text{ K)}$. Find the (a) the thermal resistance of the tube cylinder (b) temperature at the interface between the tube cylinder and the nuclear rod (c) the temperature at the outer surface of the tube cylinder. The steady-state one dimensional energy equation is given here for your reference: [25%]

$$d(k(dT/dx))/dx + q = 0 \quad \text{where } q \text{ is the internal heat generation within the material}$$

2. Water flowing at 2 kg/s through a 40-mm diameter is to be heated from 25 to 75°C by maintaining the tube surface temperature at 100°C . a) What is the required tube length for these conditions? b) What is the rate of heat transfer to the water? c) What is the pressure gradient for this flow rate? Water Properties: $C_p = 4181 \text{ J/(kg K)}$, $k = 0.643 \text{ W/(m K)}$, $Pr = 3.56$, $\mu = 547 \times 10^{-6} \text{ Ns/m}^2$. For laminar flow, $Nu = 3.66$, $f = 64/Re$ and for turbulent flow $Nu = 0.023 Re^{0.8} Pr^{0.4}$, $f = 0.184 Re^{-0.2}$, where Re is the Reynolds number and f is the Darcy friction factor. [25%]



3. Consider the two tanks shown in the following figure. Tank A initially contains salt water with 35 grams of salt per liter of solution and $h_A = 10.0$ m. Tank B initially contains pure water with $h_B = 10.0$ m. Both tanks have a diameter of 8.0 m. Find the salt concentration of tank B in grams per liter of solution at 10 s after the initial time $t = 0$. Assume that the contents of both tanks are well mixed, so the salt is uniformly distributed through out each tank. Neglect the volume of salt water below the outlet of Tank B. [25%]



4. Derive an equation for the velocity profile of a viscous fluid entrained between two long concentric cylinders. The inner radius is R_i and the outer radius is R_o . The inner cylinder is stationary and the outer cylinder rotates with a constant peripheral speed of U_o . See the following figure. [25%]

