



國立雲林科技大學

八十八學年度研究所碩士班入學考試試題

所別：機械所

科目：工程數學

1. Find the power series solution of the following initial value problem about 1.
(15%)

$$xy'' - y' + y = 0$$

$$y(1) = 2$$

$$y'(1) = -4$$

2. (a) Find an orthogonal matrix that diagonalizes the following matrix: (5%)

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

- (b) Show the result of diagonalization. (10%)

3. (a) Evaluate: (10%)

$$\oint_C (z^2 - 4z + 8) dz$$

C: the rectangle with vertices 1, 8, 8+4i, 1+4i.

- (b) Evaluate: (10%)

$$\oint_C \frac{-2iz^2}{(z+1-2i)^2} dz$$

C: any simple closed curve enclosing (-1+2i)



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4. Solve the following equations

(a) $x \frac{dy}{dx} - y = \frac{x^3}{y} e^{y/x}$ (10%)

(b) $y_1' + y_2' - y_1 = \cos 2t$, $y_1' + 2y_2' = 0$, (15%)

$$y_1(0) = y_2(0) = 0$$

5. Solve the boundary value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + qx$, $0 \leq x \leq L$

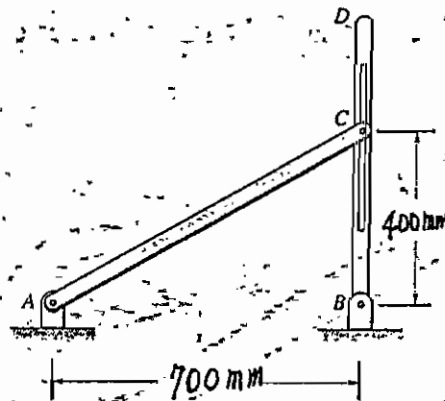
with initial condition $u(x, 0) = 0$

and boundary conditions $u(0, t) = 0$, $u(L, t) = 0$ (25%)

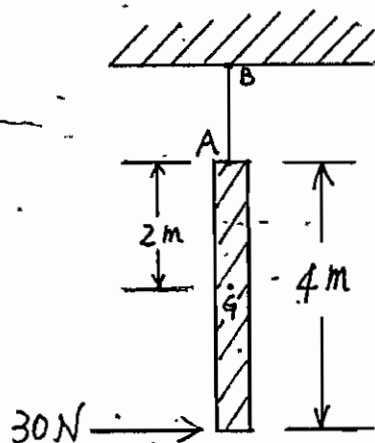


1. Bar AC has angular velocity of 2 rad/s in the counterclockwise direction that is decreasing at 4 rad/s^2 . The pin at C slides on the slot in bar BD. Consider the instant shown.

- (a). Determine the angular velocity of bar BD and the velocity of the pin relative to the slot. [12%]
 (b). Determine the angular acceleration of bar BD and the acceleration of the pin relative to the slot. [13%]



2. The slender, 200-kg beam is suspended by a cable at its end as shown. If its other end is subjected to a horizontal force of 30 N, determine the initial acceleration of its mass center G, the beam's angular acceleration, the tension in the cable AB, and the initial acceleration of the end A. [25%]

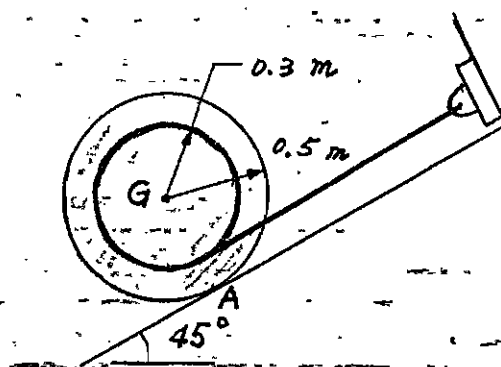




3. The spool has a mass of 80 kg and a radius of gyration $k_G = 0.4$ m. If it is released with an angular velocity of $\omega = 1$ rad/s, determine how far its center descends down the plane before it attains an angular velocity of $\omega = 10$ rad/s.

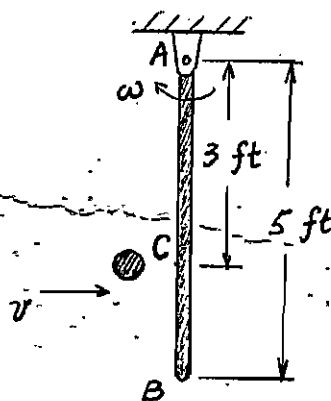
Use the energy method only.

- (a) Neglect friction and the mass of the cord, which of is wound around the central core. [10%]
 (b) Solve the problem if the coefficient kinetic friction between the spool and plane at A is $\mu_k = 0.15$. [15%]



4. The 10-lb slender rod AB is released with an angular velocity of $\omega = 0.2$ rad/s when it is in the horizontal position so that it begins to rotate clockwise. A 2-lb ball is thrown at the rod with a velocity $v = 60$ ft/s. The ball strikes the rod at C at the instant the rod is in the vertical position as shown. Take the coefficient of restitution $e = 0.7$.

- (a) Determine the angular velocity of the rod just after the impact. [12%]
 (b) What is the velocity of the ball v if the answer of (a) is 0 rad/s? [13%]
 (hint: use the principle of impulse and momentum)





1. Consider the system as shown in Fig. 1, where

$$G(s) = \frac{5}{s^2 - s - 6} \quad \text{and} \quad C(s) = k \frac{s + \alpha}{s + 10}$$

- (A) If $\alpha = 2$, sketch the root locus for $k > 0$, and determine the range of k for which the system is stable. (12%)
- (B) If $\alpha = -3$, determine the range of k for which the system is stable. (6%)
- (C) Determine all possible values of α with which at least one real value of the gain k can be found to stabilize the system. (7%)

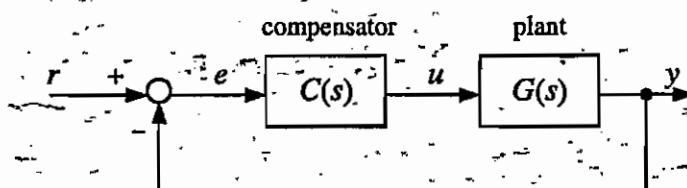


Fig. 1

2. Consider a system described by the following transfer function,

$$\frac{Y(s)}{U(s)} = \frac{s - 2}{s^2 + 3s + 2}$$

where U is the input and Y is the output.

- (A) If the input u is given by $u(t) = 3t$, determine the time response for the output, i.e., $y(t)$. (Assume $y(0) = \dot{y}(0) = 0$.) (10%)
- (B) Determine the input u as function of time by which the output satisfies $y(t) = 0$ for $t > 0$. (Assume $u(0) = 1$.) (8%)
- (C) If the input variable is constrained by

$$|u(t)| \leq 100,$$

is it possible to keep the output at a constant value at all times? Why? (7%)



(3)



(figure 2)

$$G(s) = \frac{K(1+Ts)^2}{s^3(1+s)}$$
 is given.

K & T are adjustable parameters.

Sketch the boundary between the stable and unstable regions in the K - T plane.

(20%)

(4) on the figure 2,

$$G(s) = \frac{K(1+Ts)^2}{s^3(1+s)}$$
 is given,

And, $K = 2$, $T = 3$ are given.

(15%) Of $u(t) = 2\sin(t)$, please find the steady state output of $y(t)$.

(15%) Of $u(t) = 1(t) + 4\sin(t)\cos(t)$, please find the steady state output of $y(t)$.
(where $1(t)$ is unit step function.)



- 1、試述電解加工之電極如何設計，共有 3 種方法。(15%)
- 2、試述分別說明如何使用放電加工及雷射加工法來進行材料表面處理。(10%)
試述如何將 $\phi 0.3 \text{ mm}$ 的 S U S 3 1 6 線材加工到尖端只有 $\phi 0.001 \text{ mm}$ 左右的方法(10%)。並說明加工時可能會遭遇的問題？如何解決？(10%)加工之後如何量測？可能發生的量測誤差？(5%)
3. (1) 試說明 abrasive flow machining(AFM) 之目的、加工特性、所需之相關設備與治具，並舉出幾個應用的例子。 [15 %]
(2) 試說明 rotational molding 之作業特性、流程與應用的例子。 [10 %]
4. (1) 以鋁輪圈為例，說明低壓鑄造法之作業流程。 [8 %]
(2) 試說明 friction welding 之作業流程與優缺點。 [7 %]
(3) 在金屬切削中，試說明人造鑽石與磁金刀具(cermets)之切削特性。 [10 %]



1. A pin-connected structure is loaded and supported as shown in Fig. 1. Member CD is rigid and is horizontal before the load P is applied. Member A is an aluminum alloy bar with a modulus of elasticity of 75 GPa and a cross-sectional area of 1000 mm^2 . Member B is a structural steel bar with a modulus of elasticity of 200 GPa and a cross-sectional area of 500 mm^2 . The thermal coefficients of expansion are $22 \times 10^{-6}/^\circ\text{C}$ for the aluminum alloy rod A and $12 \times 10^{-6}/^\circ\text{C}$ for the steel rod B. After the load $P = 150 \text{ kN}$ was applied, the temperature increased 100°C .
- (a) Determine the axial stresses in bars A and B. (15%)
- (b) Determine the deflection of pin D. (10%)

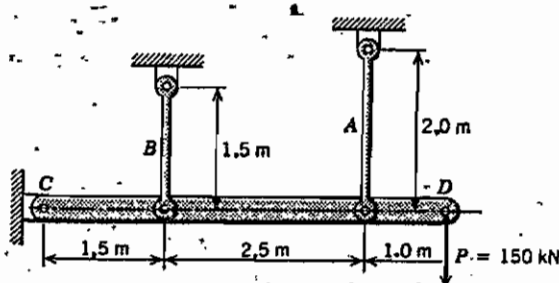


Fig. 1

2. The shaft shown in Fig. 2 is simply supported at A and D but is keyed against rotation at both points.
- (a) Find reactions at the ends, and draw a view of the element at B on the top surface of the shaft. Show all stresses acting and their numerical values. (15%)
- (b) Compute the value of the maximum shearing stress at B and the angle at which it acts. Make a view of the element properly oriented showing maximum shearing stresses acting as well as the normal stresses on all faces. (10%)

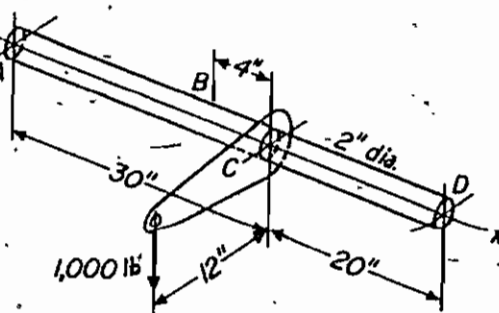


Fig. 2



3. A simple beam AB carrying two concentrated loads P (Fig. 3) has a rectangular cross section of width $b = 100$ mm and height $h = 150$ mm. The distance a from the end of the beam to one of the loads is 0.5 m. Determine the allowable value of P if the beam is constructed of wood having an allowable stress in bending $\sigma_{\text{allow}} = 11$ MPa and an allowable stress in horizontal shear $\tau_{\text{allow}} = 1.2$ MPa. Disregard the weight of the beam itself. (25%)

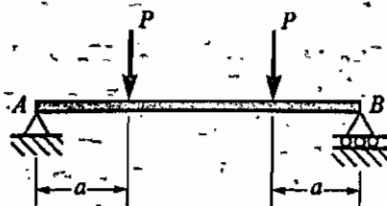


Fig. 3

4. Determine the equation of the deflection curve for a cantilever beam AB supporting a triangularly distributed load of maximum intensity q_0 (Fig. 4). Also, determine the deflection δ_b and angle of rotation θ_b at the free end. (25%)

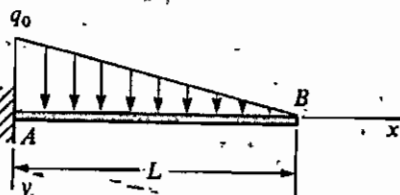


Fig. 4 Deflections of a cantilever beam with a triangular load



(25%) 1. Air inside a chamber is heated from an initial volume and pressure of 1.0 ft^3 and 1500 psia respectively to a final volume of 8.0 ft^3 . Calculate the total work done by the gas if the expansion process is quasi-static and given by the relation $PV^{1.4} = \text{constant}$.

(25%) 2. A 1 ft^3 cylinder containing an ideal gas at a pressure of 10 atm is kept in a larger tank at a pressure of 1 atm and 535°R . Calculate the work done, heat transferred and the change in internal energy if (the gas constant R is 0.73).

(a) the gas starts leaking slowly such that the cylinder and gas still remains at a constant temperature (6%)

(b) If the cylinder is insulated and process is adiabatic (6%)

(c) If the valve is suddenly opened and the tank pressure above a hypothetical piston falls to atmospheric pressure. The cylinder is insulated and the process is adiabatic (6%)

(d) If the process is polytropic with $n = 1.3$ (7%)



3. The pressure and temperature at the beginning of compression of an air-standard Diesel cycle are 95 KPa and 290 K, respectively. At the end of heat addition, the pressure is 6.5 MPa and the temperature is 2000 K. Determine
- the compression ratio, 8%
 - the thermal efficiency of the cycle, 8%
 - the mean effective pressure, in KPa. 9%
4. A cycle machine operates by interacting with a closed tank and a steady flow device. The tank has rigid walls and is initially filled with air at a pressure $P_1 = 400$ KPa and a temperature $T_1 = 300$ °C. Water flows at constant pressure through the steady-state device at a constant mass flow rate 0.5 Kg/sec. The inlet and outlet temperature of water are $T_{in} = 25$ °C and $T_{out} = 55$ °C. The operation of the cyclic machine stops when the pressure in tank drops to half of its original value, i.e., when $P_2 = 200$ KPa. Assuming that air behaves as an ideal gas with constant specific heats, $C_v = 0.72$ KJ/Kg-K and $R = 0.288$ KJ/Kg-K and water behaves as an incompressible fluid with a constant specific heat $C = 4.2$ KJ/Kg-K. Determine
- the final temperature of the air in the tank, 6%
 - the change in internal energy and entropy per unit mass of air in the tank between its initial and final states, 6%
 - the change in enthalpy and entropy per unit mass flow rate of water between the inlet and the outlet stream, 6%
 - the minimum volume of the tank that should be used to produce a work $W = 200$ KJ from the cyclic machine. How long would this process take? 7%



(1) Consider the laminar flow of an incompressible and constant property fluid flows in a tube with the tube radius of r_0 at the fully developed condition. Determine (a) the velocity profile, $u(r)$. The momentum equation for the laminar fluid flow in tube is giving as:

$$(\mu/r)d(rdu/dr)/dr = dp/dx,$$

where the pressure gradient, dp/dx , is constant, μ is the fluid viscosity.

(b) Calculate the pressure drop per unit length, dp/dx , in the fully developed region for the laminar flow of 20°C water through a tube with the radius $r_0 = 1.35$ cm, water density, $\rho = 1000$ kg/m³, viscosity, $\mu = 1.01 \times 10^{-3}$ Ns/m². The mass flow rate of the stream is $M = 123.72$ kg/hr. (25%)

(2) A constant property liquid is kept within two large horizontal-parallel plates in which the upper plate is moving with a constant speed U , while the lower plate is maintained stationary. The vertical distance between the two parallel plates is L . The upper plate is maintained at a constant temperature of T_L and the lower plate is insulated. (a) Determine the temperature of the insulated plate, expressing your result in terms of fluid properties and the temperature and speed of the moving plate. (b) Obtain an expression for the heat flux at the moving plate. Assuming the liquid velocity is linear distribution between the plates. The energy equation is given as:

$$\rho c_p (u \partial T / \partial x + v \partial T / \partial y) = \partial (k \partial T / \partial x) / \partial x + \partial (k \partial T / \partial y) / \partial y + \partial (\mu \partial T / \partial y) / \partial y$$

You can simplify the above equation for obtaining the solution. (25%)



(3) The concentric cylinder viscometer shown in Fig.1 uses a falling weight W to produce a constant rotational speed of the inner cylinder. Find the require weight if the inner cylinder rotates at 30 rpm and the fluid in the viscometer is gasoline at 50°C ($\mu=2.39\times 10^{-3}\text{ N}\cdot\text{s}/\text{m}^2$) (25%)

(4) Experimental measurements are made in a low-speed air jet to determine the drag force on a circular cylinder. Velocity measurements at two sections, where the pressure is uniform and equal, give the results shown in Fig.2. Evaluate the drag force on the cylinder, per unit width. Express your result using the parameters shown in the figure.

($u=U\sin(\frac{\pi y}{2a})$, $0\leq y\leq a$; $u=U$, $y>a$)(25%)

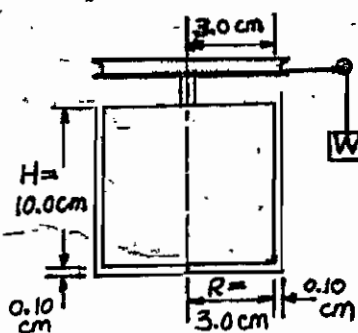


Fig.1

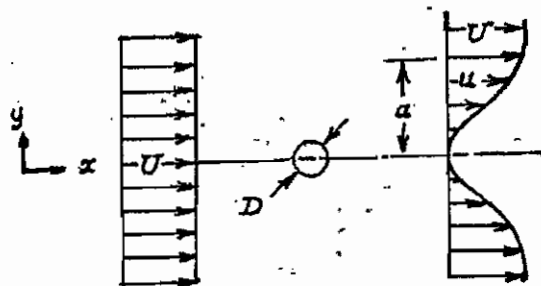


Fig.2



1. Find the arc length of the graph of $y = x^3/6 + 1/(2x)$ on the interval $[1/2, 2]$. (25%)
2. A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring an additional 3 inches. (25%)



(3) $\lim_{x \rightarrow 0} \frac{e^x}{\sin(x)}$

Does the limit shown above exist?

If it does, please find it. (10%)

(4) $f(x) = x^3 - 3x^2 + 3x$, $x \in \mathbb{R}$ is given.

Does there exist local maximum or minimum points?

If there does, please find all of them. (20%)

(5) 兩個變動的同心圓，兩圓之間所圍的面積恆為 9π ，
大圓面積之變化率為 $10\pi \text{ cm}^2/\text{sec}$ 。

當小圓之面積為 $16\pi \text{ cm}^2$ 時，其圓周之變化率為何？
(20%)



注意：將依說明的完整性及所敘述系統功能的強弱，作為評分的依據。

1. 以您所瞭解的最先進的個人電腦為例，試詳細說明個人電腦的組成及其各部的功能。並以實例說明您使用電腦的經驗。 (10%)
2. 請詳細說明下列問題 (10%)
 - (1) 何謂 INTERNET? 何謂 WWW?
 - (2) 台灣地區有那些 INTERNET 網路?
 - (3) 上 INTERNET 所需的軟硬體配備為何?
 - (4) 上 INTERNET, 我們能做什麼?
 - (5) 何謂 TCP/IP? 何謂 HTML? 何謂 Home Page?
3. 以您所瞭解的自動化系統，試舉一例說明它的組成及其各部的功能。(請特別著重能導致系統自動化運轉的各子系統之功能說明) (10%)
4. 以精密加工為例，試說明您所瞭解的最高精度的加工實例。(詳細說明成品的材料、尺寸、精度加工的机具、刀具；加工參數及加工流程，---) (10%)
5. 何謂解析度？試以電腦顯示器 (Monitor) 的解析度為例，予以說明。可動的機械系統，其運動是否解析度？試以實例予以說明，並明示決定運動解析度的零組件及其形成原因。 (10%)



6. 電爐常用 PID 控制器控制其溫度,說明 PID 控制器的原理。(10%)
7. 鋼板工場的扁鋼胚加熱爐是使用重油加熱的動樑式加熱爐,試述如何增加其能源效率。(10%)
8. 汽車的油箱殼是冷軋鋼片的深抽成型品,當深抽過程發生油箱殼破裂,試述探討分析油箱殼破裂的步驟及其可能原因。(10%)
9. 材料在加熱時,長度會發生變化,設計一實驗裝置,說明如何量測長度對溫度($25 \sim 1000^{\circ}\text{C}$)的變化。(10%)
10. 晶圓(wafer)是半導體工業的重要材料,一座 8 吋晶圓工場從拉單晶到製成晶圓,要經過那些製程,需要檢驗那些項目,詳細說明。(10%)