



本份試卷共 5 大題計算問答題，未提供計算過程或說明者不計分。

1. (20 points) Consider the following model with  $\theta < 1$ .

The quantity of output produced at time  $t$  is:

$$Y(t) = A(t) \cdot (1 - a_L)L(t)$$

where,

$Y$  denotes output,

$A$  denotes technology,

$L$  denotes labor,

$a_L$  is the fraction of the labor force used in the R&D sector, and  $1 - a_L$  is the fraction of labor force used in the goods-producing sector.  $a_L$  and  $1 - a_L$  are exogenous and constant.

The production function for new knowledge is:

$$\dot{A}(t) = B \cdot [a_L \cdot L(t)]^\gamma \cdot A(t)^\theta, \quad B > 0, \quad \gamma \geq 0$$

where  $B$ ,  $\gamma$ , and  $\theta$  are parameters.

In addition, population growth is exogenous. Thus,

$$\dot{L}(t) = n \cdot L(t), \quad n \geq 0$$

where  $n$  is a constant.

- (a) On the balanced growth path,  $\dot{A} = g^* A \cdot A(t)$ , where  $g^*$  is the balanced growth-path value of  $g_A$  and  $g_A$  is the growth rate of  $A$ . Use this fact to derive an expression for  $A(t)$  on the balanced growth path in terms of  $B$ ,  $a_L$ ,  $\gamma$ ,  $\theta$ ,  $n$ , and  $L(t)$ .
- (b) Use your answer to part (a) and the production function described above, to obtain an expression for  $Y(t)$  on the balanced growth path. Find the value of  $a_L$  that maximizes output on the balanced growth path.

2. (20 points) Suppose that output at firm  $i$  is given by  $Y_i = K_i^\alpha \cdot L_i^{1-\alpha} \cdot (K^\phi \cdot L^{-\phi})$ . Here  $K_i$  and  $L_i$  are the amounts of capital and labor used by the firm;  $K$  and  $L$  are the aggregate amounts of capital and labor; and  $\alpha > 0$ ,  $\phi > 0$ , and  $0 < \alpha + \phi < 1$ . Assume that factors are paid their private marginal products; thus  $r = \partial Y_i / \partial K_i$ . Assume that the dynamics of  $K$  and  $L$  are given by  $\dot{K} = s \cdot Y$  and  $\dot{L} = n \cdot L$ , and that  $K_i / L_i$  is the same for all firms.  $s$  and  $n$  are constants.



- (a) What is  $r$  as a function of  $K/L$ ?
- (b) What is  $K/L$  on the balanced growth path? What is  $r$  on the balanced growth path? (i.e., derive an expression for  $K/L$  and an expression for  $r$  on the balanced growth path in terms of  $\alpha$ ,  $\phi$ ,  $s$ , and  $n$ .)
3. (10 points) Consider two economies (indexed by  $i=1,2$ ) described by  $Y_i(t) = K_i(t)^\theta$  and  $\dot{K}_i(t) = s_i \cdot Y_i(t)$ , where  $\theta > 1$ . Suppose that the two economies have the same initial value of  $K$ , but that  $s_1 > s_2$ . Prove analytically that  $Y_1/Y_2$  is continually rising.
4. (20 points) 請用 30 字以內的字數解釋以下名詞，請勿畫圖或使用任何數學符號及公式。
- (a) Price discrimination
- (b) Pareto efficiency
- (c) Public good
- (d) Substitution effect
- (e) Income elasticity of demand
5. (30 points) 政府預定於近期內開徵特種貨物及勞務稅(奢侈稅)，將針對持有不動產未滿一年(兩年)的出售人，就銷售價格課徵 15%(10%)的特種銷售稅。
- (a) 目前奢侈稅定位為銷售稅，也就是不論賣方出售時有沒有賺錢都要繳稅。假設投資客老王以 2,000 萬元買進台北市大安區的一戶公寓，目前持有未滿一年，另一投資客老張願出 2,300 萬元購買，請問老王會不會賣?
- (b) 呈上題，假設老王目前願意用 2,500 萬賣給老張，由目前市場狀況判斷一年內該公寓會漲到 2,800 萬，請問若預期奢侈稅即將開徵，老張會不會買?
- (c) 請用圖形分析政府對賣方(出售人)課稅對於房地產市場的供需及價量的影響。(為簡化起見，你的圖形中可以假設奢侈稅為從量稅)
- (d) 若政府改向不動產的買方課稅，請問你在(c)小題的答案會不會不同? 為什麼?
- (e) 有人覺得應該是賣方出售時有賺錢才應課稅，也就是只針對資本利得課徵奢侈稅。請問你覺得哪一種方式比較能夠有效的抑制房價? 為什麼?