國立雲林科技大學
99 學年度博士班招生考試試題

系所別：財務金融系
科　目：微積分

1. 設 $f(x) = \dfrac{x-1}{x+1}$，$x \in [1, 2]$。試在（1,2）開區間中，求 x_0 使適合均值定理中之結論。 （10分）

2. 設圓之半徑為 r，試證明其面積為 πr^2。 （10分）

3. 試求 $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)x^n}{n!}$ 之收斂區間。 （10分）

4. 某公司擬撥出 \$10,000 做商品廣告，電視廣告每分鐘需成本 \$3,000，而電台廣告每分鐘需成本 \$1,000。若公司購買電視廣告 x 分鐘，電台廣告 y 分鐘，公司之收益（以 \$1,000 為單位）為 $f(x,y) = -2x^2 - y^2 + xy + 8x + 3y$，試問公司應如何做廣告使公司的收益為最大？ （10分）

5. 試計算 $I = \displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right) dy\,dx$。 （10分）
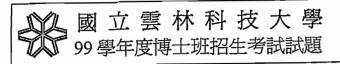
6. 試證明級數 $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^{n-1}}{(2n^2 + n + 1)^{\frac{n+3}{2}}}$ 收斂。 （10分）

7. 設 $f(x) = (1+x)^x - 1$，其中 $x > 0$。求 f 的導函數 $f'(x)$。 （10分）

8. $\displaystyle\lim_{x \to -2} \dfrac{\sqrt[3]{x-25} + 3}{\sqrt[4]{x+3} - 1} = ?$ （10分）

9. $\displaystyle\lim_{x \to 0} \dfrac{1 - \cos 4x}{x^2} = ?$ （10分）

10. 設 $u = \dfrac{x+z}{y}$，其中 $ze^z = xe^x + ye^y$。求 $\dfrac{\partial u}{\partial x}$。 （10分）

本試題共四大題，每大題 25 分，合計 100 分。請依題目順序將答案寫在答案卷上，違者不予計分。

1. a. Please explain the meaning of Weak Axiom of Revealed Preference(WARP) and Strong Axiom of Revealed Preference(SARP).

   b. Consider the following data which is a consumer's choice when he faces different price sets:

   A: $(p_1,p_2,p_3) = (1,3,10)$　and　$(x_1,x_2,x_3) = (3,1,4)$

   B: $(p_1,p_2,p_3) = (4,3,6)$　　and　$(x_1,x_2,x_3) = (2,5,3)$

   C: $(p_1,p_2,p_3) = (1,1,5)$　　and　$(x_1,x_2,x_3) = (4,4,3)$

   Does the consumer violate SARP and WARP? Please explain in detail.

2. a. A monopolist is operating at an output level where demand elasticity $|\varepsilon|=3$. The government imposes a quantity tax of $6 per unit of output. If the demand curve facing the monopolist is linear, how much does the price rise?

   b. What is the answer to the above question if the demand curve facing the monopolist has constant elasticity?

3.　Please answer the following questions about labor market.

a. What are the definitions of <u>labor force</u> and <u>employed population</u> in Taiwan? (6%)

b. Please explain how to calculate <u>labor force participation rate</u> and <u>unemployment rate</u>? (4%)

c. In Taiwan, male's labor participation rate has decreased about 10% over the last three decades. On the contrary, female's labor participation increased 10% during the same period. Please provide at least <u>three</u> explanations for this phenomenon. (9%)

d. Let U be the total unemployed population, and E be the total employed population. Define job separation rate (s) as the percentage of population in E who quit their job and define job finding rate (f) as the percentage of population in U who find a job in a given period of time. Under the above definition, we can express the change in employment population in the given period of time (ΔU) as

$$\Delta U = sE - fU.$$

Lastly, we define the natural rate of unemployment as the unemployment rate such that $\Delta U = 0$. Please show that the natural rate of unemployment is equal to $s/(s+f)$. (6%)

4.　Please answer the following questions about monetary market and inflation.

a. Please show the definition of M_1B and M_2 in Taiwan. (6%)

b. You are given the following macroeconomic data of country A: the GDP growth rate is 4%; the money supply growth rate is 6%; the nominal interest rate is fixed at 3%. Suppose the estimated money demand function in country A is $(M^d/P) = K*(Y/R)$, where P is the price level; Y is the real GDP; R is a fixed nominal interest rate; and K is a constant. Please find the inflation rate in country A. (6%)

c. The above question continued. Suppose the new president of central bank in country A predicts that the GDP growth rate in the near future will decrease to 1%. To remain a stable inflation rate after the slowdown of GDP growth, what the level of the money supply growth rate should he set? (6%)

d. Base on his research, Milton Friedman, a famous economist and Nobel Prize winner concluded that: *Inflation is always and everywhere a monetary phenomenon.* Can you use an IS-LM framework to show Friedman's idea? If yes, please show how. If no, please explain why? (7%)