



每題 10 分

1. Prove that  $(x^n - y^n)$  has  $(x - y)$  as a factor for all positive integers  $n$ .
2. Prove that if  $\lim_{n \rightarrow \infty} u_n$  exists, it must be unique.
3. Show that  $\int_1^2 \frac{dx}{(x^2 - 2x + 4)^{\frac{3}{2}}} = \frac{1}{6}$ .
4. If  $T = x^3 - xy + y^3$ ,  $x = \rho \cos \phi$ ,  $y = \rho \sin \phi$ , find  $\partial T / \partial \rho$ .
5. For what value of  $x$  does  $\sum_{n=1}^{\infty} n!(x-a)^n$  converge?
6. Evaluate the limit  $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{1}{x}}$ .
7. A graph satisfies the equation  $x^3 + 3xy - 2y - xy^2 + 1 = 0$ . Find the tangent line at the point  $(1, 2)$  of the graph.
8. Solve the differential equation  $\frac{dy}{dx} = \frac{2x-1}{x^2-x+1}$  with  $y(0) = 0$ .
9. Use a second-degree Taylor polynomial centered at 0 to approximate the integral  $\int_0^1 \frac{1+e^{-x}}{e^x+e^{-x}} dx$ .
10. A manufacturer's production is modeled by the function  $f(x, y) = 100x^{0.4}y^{0.6}$  where  $x$  represents the units of labor and  $y$  represents the units of capital. Each capital unit costs \$150 and each labor unit costs \$300. The total expenses for labor and capital cannot exceed \$7,500. Find the maximum production level.



注意事項：本試題共有四大題，每大題分數各為 25 分。其中，第 1、2 題係個體經濟學題目，第 3、4 題係總體經濟學題目。

1. Mary has the utility function  $U(x,y)=y/(100-x)$ 
  - (i) Does Mary prefer more to less of both goods? (7 points)
  - (ii) Draw a diagram showing Mary's indifference curves corresponding to the utility level  $U=1/2$ ,  $U=1$ , and  $U=2$ . (6 points)
  - (iii) Please describe the set of indifference curves for Mary. (6 points)
  - (iv) If the price of  $x$  is \$1 and the price of  $y$  is \$1, find Mary's demand for  $x$  as a function of her income and draw a diagram showing her Engel curve for  $x$ . (6 points)
  
2. A competitive firm has a production function described as follows. "Weekly output is the square root of the minimum of the number of units of capital and the number of units of labor employed per week." Suppose that in the short run this firm must use 16 units of capital but can vary its amount of labor freely.
  - (i) Write down a formula that describes the marginal product of labor in the short run as function of the amount of labor used. (Be careful at the boundaries.) (7 points)
  - (ii) If the wage is  $w=\$1$  and the price of output is  $p=\$4$ , how much labor will the firm demand in the short run? (6 points)
  - (iii) What if  $w=\$1$  and  $p=\$10$ ? (6 points)
  - (iv) Write down an equation for the firm's short-run demand for labor as a function of  $w$  and  $p$ . (6 points)



3. The Small Land is a closed economy and obeys our short-run IS-LM model.

Assume it starts out in equilibrium in both the goods market and the money market.

The Small Land's economy is described by the following set of equations:

Goods market:

- $C = c_0 + c_1(1-t)Y$ , where  $C$  is consumption;  $Y$  is income;  $t$  represents a proportional tax; and  $c_0$  and  $c_1$  are positive constants.
- $I = b_0 - b_1i$ , where  $I$  is investment;  $i$  is the interest rate; and  $b_0$  and  $b_1$  are positive constants.
- $G = \bar{G}$ , where  $\bar{G}$  is a positive constant.

Money market:

- $M^d = P(m_0 + m_1Y - m_2i)$ , where  $M^d$  is money demand;  $P$  is the price level;  $m_0$  (a positive constant) represents exogenous changes to  $M^d$ ; and  $m_1$  and  $m_2$  are also positive constants.
  - Let  $M^s$  represent money supply.
- (i) Combine the goods market equations to derive an expression for  $Y$  as a function of  $i$  (i.e. derive the IS curve). Give the definition for why the IS curve slopes upward/downward. (5points)
  - (ii) Use the money market equations to express  $i$  as a function of  $Y$  (i.e. derive the LM curve). Give intuition for why the LM curve slopes upward/downward. (5points)
  - (iii) Suppose the government increases its spending by  $\Delta \bar{G}$ . Which curve will shift, if any? Calculate by how much it will shift and draw a diagram that shows the impact of this policy. (5points)
  - (iv) What will happen to investment as a result of the government policy described in part (iii)? (You do not need to calculate anything, just give intuition.) (5points)
  - (v) Suppose that the government decides to cut taxes instead of increasing spending. Analyze the effects of this expansionary fiscal policy using a diagram. (You do not need to calculate anything, just draw the diagram.) (5points)



4. Suppose that a closed economy has the following Characteristics:

- Wage Setting Relation:  $W = P^e F(u, z)$ , where  $u$  is the rate of unemployment  
and  $z$  is a variable affecting  $W$  except  $u$ .
  - Price Setting Relation:  $P = (1 + \lambda) W$ , where  $\lambda$  is a constant.
  - Goods Market:  $Y = C(Y, T) + I(Y, i) + G$ ,
  - Financial Market:  $M^s = M^d(Y, i) P$ .
- (i) Find the aggregate supply relation. Describe the channel through which the AS curve slopes up/down. (6 points)
  - (ii) Assume that the economy is at a point such that the unemployment rate is equal to the natural rate of unemployment. What does this imply about the price level and output? Explain. (6 points)
  - (iii) If the Fed carries out a monetary contraction, what happens in the short-run and the medium-run/long-run? Start from point A where  $P = P^e$ . (7 points)
  - (iv) What does neutrality of money imply about the effectiveness of contractionary monetary policy in affecting output in the short- and medium-run/long-run? Define neutrality of money. (6 points)