國立雲林科技大學
97學年度博士班招生考試試題

系所別：財務金融系
科　目：經濟學

There are four questions included in the exam. The first two questions are <u>microeconomics</u> and the latter two are <u>macroeconomics</u>. The weight points for each question are assigned.

1. Consider a profit-maximizing firm with the production function $y = f(x_1, x_2)$, facing output price p and factor prices w_1 and w_2. Suppose this firm is taxed according to the total cost of factor 2, i.e., tax = tw_2x_2.
   (a) Derive the factor demand functions. Are these choice functions homogeneous of any degree in any of the parameters? *(7points)*
   (b) Show that if the tax rate rises, the firm will use less of factor 2. *(6points)*
   (c) Show that $\partial x_1^* / \partial t = w_2 \partial x_2^* / \partial w_1$ *(6points)*
   (d) Suppose that factor 1 is held fixed at its profit-maximizing level. Show that the response of factor 2 to a change in the tax rate is less in absolute value than before. *(6points)*

2. Find the production function associated with each of the following cost functions:
   (a) C=$\sqrt{w_1 w_2}\ e^{y/2}$ *(9points)*
   (b) C=$w_2[1 + y + \log(w_1 / w_2)]$ *(8points)*
   (c) C=$y(w_1^2 + w_2^2)^{1/2}$ *(8points)*

3. Suppose that planned expenditure is given by $E = C(Y-T) + I(r) + G$, where E=expenditure, $C(.)$=consumption function, Y=GDP, T=tax revenues, $I(.)$=investment function, r=interest rate, and G=government expenditure.
   (a) Show how equal increases in G and T affect the position of IS curve. That is, what is the effect on Y for a given level of r? *(8points)*
   (b) Show how equal increases in G and T affect the position of AD curve. That is, what is the effect on Y for a given level of inflation rate, π? *(9points)*
   【Hints: Substitute LM curve $r = r(Y, \pi)$ into the IS curve to find the effect on Y】
   (c) Suppose that tax revenues T, instead of being exogenous, are a function of GDP: $T = T(Y)$, $T'(Y) > 0$. Show how an increase in $T'(Y)$ affects the slope of the IS curve. *(8points)*

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4. Consider an individual who lives for two periods and has constant-absolute-risk-aversion utility, $U = -e^{-\gamma C_1} - e^{-\gamma C_2}$, $\gamma > 0$. The interest rate is zero and the individual has no initial wealth, so the individual's life budget constraint is $C_1 + C_2 = Y_1 + Y_2$.

Y_1 is certain, but Y_2 is normally distributed with mean $\overline{Y_2}$ variance σ^2.

(a) With an instantaneous utility function $U(C) = -e^{-\gamma C}$, $\gamma > 0$, what is the sign of $U'''(C)$? *(8points)*

(b) What is the individual's <u>expected</u> lifetime utility as a function of C_1 and the exogenous parameters Y_1, $\overline{Y_2}$, σ^2, and γ? 【Hints: If a variable x is normally distributed with mean μ and variance V, $E[e^x] = e^{\mu} e^{V/2}$】 *(9points)*

(c) Find an expression for C_1 in terms of Y_1, $\overline{Y_2}$, σ^2, and γ. *(8points)*

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C2

每題 10 分

1. Solve for $x: 5xe^{-x} + x^2e^{-x} = 0$.
2. Find y' and the slope of tangent line to the graph of $3e^x + e^{3y} = 4e^x + y$ at the point (0,0).
3. Find the particular solution for the differential equation $\frac{dy}{dx} = \sqrt{x}$; $y(4) = 0$.
4. Find the Taylor series of $\frac{1}{1+27x^3}$ at 0 and state the interval of convergence.
5. Find the value of $\int_{-\infty}^{\infty} f(x)dx$ if it converges.

$$f(x) = \begin{cases} 1 + x^3 & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

6. Use the second derivative test to find the relative maxima and minima of the function

$$f(x) = 2x^3 + 3x^2 - 12x - 7$$

7. The probability density function for the duration of telephone calls in a certain city is

$$f(x) = \begin{cases} 0.5e^{-0.5x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Where x denotes the duration (in minutes) of a randomly selected call. Find the probability that a randomly selected call will last between 2 and 3 minutes.

8. Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}}$$

9. Use a Taylor polynomial of degree 8 to approximate

$$\int_0^1 e^{-x^2} dx$$

10. Two sides of a triangle are 4 inches long. What should the angle between these sides be to make the area of the triangle as large as possible?

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