## 國立雲林科技大學 96 學年度博士班招生入學考試試題

系所別:財務金融系 科 目:經濟學

There are four questions included in this test. The weight of each question is equally 25 points. Each question may include a couple of sub-questions. Their weights are shown in points.

- 1. A factor of production i is called an inferior if the conditional demand for that factor decreases as output increases; that is,  $\partial x_i(\mathbf{w}, y)/\partial y < 0$ , where w is vector of factor
  - (a) Draw a diagram indicating that inferior factors are possible. (7 points)
  - (b) Show that if the technology is constant returns to scale, then no factors can be inferior. (9 points)
  - (c) Show that if marginal cost decreases as the price of some factor increases, then the factor must be inferior. (9 points)
- 2. Suppose marginal costs are constant at c>0 and that the demand function is given by

$$D(p) = \begin{cases} 10/p & \text{if } p \le 20\\ 0 & \text{if } p > 20 \end{cases}$$

What is the profit-maximizing price?

(25 points)

- 3. Consider the multiplier-accelerator model of income determination:
  - (1) Consumption depends on the previous period's income:  $C_t = a + bY_{t-1}$ .
  - (2) The desired capital stock is proportional to the previous period's output:

$$K_{t}^{*}=cY_{t-1}.$$

- (3) Investment equals the difference between the desired capital stock and the stock inherited the previous period:  $I_t = K_t^* K_{t-1} = K_t^* cY_{t-2}$ .
- (4) Government purchases are constant:  $G_i = \overline{G}$ .

$$(5) Y_{i} = C_{i} + I_{i} + G_{i}.$$

- (a) Express  $Y_i$  in terms of  $Y_{i-1}$ ,  $Y_{i-2}$  and the parameters of the model. (10 points)
- (b) Suppose b=0.9 and c=0.5. Suppose there is a one-time disturbance to government purchases; specifically, suppose that G is equal to  $\overline{G}+1$  in period t and is equal to  $\overline{G}$  in all other periods. How does this shock affect output in future different 5 periods? (15 points)



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4. Assume that an economy has a production function specified as

$$Y = F(K, AL),$$

where Y = output,

K = physical capital,

A =knowledge, and

L = labor.

The changesin factors per year are respectively as follows.

$$\dot{K} = sY(t) - \delta K(t),$$

$$\vec{L} = nL(t),$$

$$\overset{\bullet}{A} = gA(t),$$

Where  $s, \delta, n, g$  are all constant.

Suppose that both labor and physical capital are paid by their marginal products.

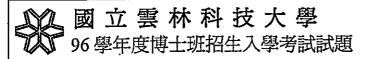
Let 
$$w = \frac{\partial F(K, AL)}{\partial L}$$
 and  $r = \frac{\partial F(K, AL)}{\partial K} - \delta$ , where  $w =$  wage and  $r =$  interest rate.

(a) If the production function can be written in another way as

$$y = f(k)$$
, where  $y = \frac{Y}{AL}$  and  $k = \frac{K}{AL}$ . Show  $w = A[f(k) - kf'(k)]$ . (6 points)

- (b) Since constant returns to scale imply that the total amount paid to the factors of production equals total net output, show that under constant returns to scale, (7 points)  $wL + rK = F(K, AL) - \delta K$ .
- (c) Derive what the growth rates of w and r on a balanced growth path are.

(12 points)



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科 目:微積分

## Answer the following questions carefully

- 1. Find Taylor's series for  $f(x)=\sin x$  at  $x=\frac{\pi}{4}$ . (10 points)
- 2. Use Maclaurin series to approximate numerical value for  $\int_{0}^{0.1} e^{-x^2} dx$ . (10 points)
- 3. Let  $f(x) = ax^2 + bx + c$  a > 0. Prove that  $f(x) \ge 0$   $\forall x \iff b^2 4ac \le 0$ . (15 points)
- 4. Evaluate  $\lim_{x \to (\frac{\pi}{2})^{-}} (\sec x \tan x)$ . (10 points)
- 5. Prove that  $\lim_{x\to 0} x \sin \frac{1}{x} = 0$ . (15 points)
- 6. Find the derivative of the function  $f(x) = \frac{x}{e^x + 1}$  (10 points)
- 7. The plan x + y + z = 12 intersects the paraboloid  $z = x^2 + y^2$  in an ellipse. Find the highest and lowest points on this ellipse (10 points).
- 8. Use the law of logarithms to solve the equation  $e^{x/3} = 4$  (10 points)
- 9. Evaluate the improper integral  $\int_0^\infty (x-2)e^{-x^2+4x+3}dx$  if it is convergent (10 points)