



1. Choose a constant  $\alpha$  so that the differential equation is exact, then produce a potential function and obtain the general solution. (15%)

$$3x^2 + xy^\alpha - x^2 y^{\alpha-1} y' = 0$$

2. Find the general solution of the differential equation, using any method. (15%)

$$x^2 y'' + 3xy' + y = 9x^2 + 8x + 5$$

3. Solve  $y'' + 2y' + 2y = \delta(t - 3)$ ;  $y(0) = y'(0) = 0$  (10%)

4. If  $f(t) = \cos t + e^{-2t} \int_0^t f(\alpha) e^{2\alpha} d\alpha$ , find  $f(t)$ . (10%)

5. Solve the O.D.E.  $x'' - 4x' + 3x = e^t$ . (15%)

6. Suppose that the temperature  $T(K)$  at the point  $(x, y, z)$  is given by  $T = x^2 - y^2 - xyz + 273$ ; then in which direction is temperature increasing most rapidly at the point  $(1, 2, 3)$ , and what is the rate? (20%)

7. Find the eigenvalues and eigenvectors of matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

(15%)