



1. (10%) Solve the initial value problem $\frac{dy}{dx} = (-2x + y)^2 - 7, y(0)=0$.
2. (15%) Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$
3. (10%) Solve $f(t) = 3t^2 - e^{-t} - \int_0^t f(\tau) e^{t-\tau} d\tau$ for $f(t)$.
4. (15%) Evaluate $\iint_S xz^2 ds$, where S is that portion of the cylinder $y = 2x^2 + 1$ in the first octant bounded by $x=0, x=2, z=4$ and $z=8$.
5. (15%) Let $i = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$, $j = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ and $k = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$, show that
 - (a)(5%) $i^2 = j^2 = k^2 = -I$, where I is the unit matrix.
 - (b)(10%) $ij = -ji = k, jk = -kj = i, ki = -ik = j$
6. (15%) A function $f(x) = \begin{cases} -\cos(x), & -\pi \leq x \leq 0 \\ \sin(x), & 0 < x \leq \pi \\ 0, & |x| > \pi \end{cases}$, find Fourier Integral of $f(x)$
7. (10%) Please solve $x^2 y'' + xy' - y = 2(x - x^{-1})$, where $x > 0$
8. (10%) Suppose that the temperature $T(K)$ at the point (x, y, z) is given by $T = x^2 - y^2 + xyz + 273$; in which direction is temperature increasing most rapidly at $(-1, 2, 3)$, and what is the rate?