



1. Solve the following Laplace Transform $L[f(t)]$: (25%)

$$f(t) = \begin{cases} 2t, & 0 < t < 1 \\ t, & 1 < t \end{cases}$$

2. Find the solutions of the following ordinary differential equations:

(a) $y' \cdot \sin y + \sin x \cdot \cos y = \sin x$ (10%)

(b) $2xy \, dx + (4y + 3x^2) \, dy = 0$ (15%)



Prob. 3 (20%)

Given $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, Please find A^N for a given positive integral N .

Prob. 4 (10%)

Please compute the directional derivative of the field $u(x, y, z) = x^2 - 3yz$ in the direction of vector $\vec{v} = \vec{i} + \vec{j} - 2\vec{k}$ at the point $(2, -1, 4)$.

Prob. 5 (20%)

Please find the area of the given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.