

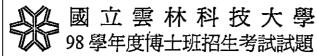
所別:產業精密機械研究所 科目:工程數學 (4)

1. Solve for the following equations.  
(a) 
$$y' - xe^{x}y^{2} = 0$$
 ( $y' = \frac{dy}{dx}$ ) (10%)  
(b)  $D^{3}y + 2Dy + y = x^{2} + \sin x + e^{2x}$  ( $D = \frac{d}{dx}$ ) (15%)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 , \quad 0 < x < a, \quad 0 < y < b$$

$$\frac{\partial u}{\partial x}|_{x=0} = 0 , \quad \frac{\partial u}{\partial x}|_{x=a} = 0, \quad 0 < y < b$$

$$u(x,0) = 0 , \quad u(x,b) = f(x), \quad 0 < x < a$$
(b) what if  $f(x) = 100$  (5%)



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## 3. (20%)

Please find the value of line integral  $\int_C (y^2 - 6xy + 6)dx + (2xy - 3x^2)dy$  along the given curve  $2^y = x^4$  from point (-1, 0) to point (2, 4)

4. (10%)

A sphere is given by  $\ln(x^2+y^2) - z^2 = 0$ . Please find the equation of tangent plane at the point  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ .

## 5. (20%)

Please find a 3x3 symmetric matrix that has eigenvalues  $\lambda_1 = 1$ ,  $\lambda_2 = 3$ ,  $\lambda_3 = 5$ , and corresponding eigenvectors  $\vec{v}_1 = [1, -1, 1]^T$ ,  $\vec{v}_2 = [1, 0, -1]^T$ ,  $\vec{v}_3 = [1, 2, 1]^T$ .