



1. Solve for the following equations.

(a)  $y' - xe^x y^2 = 0$  ( $y' = \frac{dy}{dx}$ ) (10%)

(b)  $D^3 y + 2Dy + y = x^2 + \sin x + e^{2x}$  ( $D = \frac{d}{dx}$ ) (15%)

2. (a) Solve for the following equation. (20%)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < a, \quad 0 < y < b$$

$$\frac{\partial u}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial u}{\partial x} \Big|_{x=a} = 0, \quad 0 < y < b$$

$$u(x, 0) = 0, \quad u(x, b) = f(x), \quad 0 < x < a$$

(b) what if  $f(x) = 100$  (5%)



3. (20%)

Please find the value of line integral  $\int_C (y^2 - 6xy + 6)dx + (2xy - 3x^2)dy$  along the given curve  $2^y = x^4$  from point  $(-1, 0)$  to point  $(2, 4)$

4. (10%)

A sphere is given by  $\ln(x^2 + y^2) - z^2 = 0$ . Please find the equation of tangent plane at the point  $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ .

5. (20%)

Please find a 3x3 symmetric matrix that has eigenvalues  $\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 5$ , and corresponding eigenvectors  $\vec{v}_1 = [1, -1, 1]^T, \vec{v}_2 = [1, 0, -1]^T, \vec{v}_3 = [1, 2, 1]^T$ .