國立雲林科技大學
96學年度博士班招生入學考試試題

所別：產業精密機械研究所
科目：工程數學

1. (20%)

A two-dimension wave equation in Cartesian coordinate $u(x,y,t)$ is shown below

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$ where c is constant parameter.

Please express it in polar coordinate $u(r,\theta,t)$, with

$r = \sqrt{x^2 + y^2}$ and $\theta = Tan^{-1}(y/x)$

2. (30%)

The position of a moving particle is given by $\vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.

Please find the tangent and normal components of the acceleration vector at any time t, and find the curvature

3. (15%) Consider the initial value problem

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=0,\quad x(0)=1,\quad x'(0)=c,$$

in which c is a constant. Please determine the value of c such that $x(t)=e^{-t}$ for all $t\geq 0$.

4. (15%) Consider the initial value problem

$$\frac{dx}{dt}+x=t^{10},\quad x(0)=10.$$

Please find a function $f(t)$ such that $x(t)=f(t)+\int_0^t e^{-\tau}(t-\tau)^{10}d\tau$.

5. (10%) Please expand $f(t)=\sin(t),\ -2\pi<t<2\pi,$ in a complex Fourier series.

6. (10%) Please plot the amplitude spectrum of the periodic wave that is the periodic extension of the function $f(t)=\sin(t),\ -\pi<t<\pi$.