所別:產業精密機械研究所

科目:工程數學

## 1. (20%)

A two-dimension wave equation in Cartesian coordinate u(x,y,t) is shown below

$$\frac{\partial^2 u}{\partial t^2} = c^2 (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$
, where c is constant parameter.

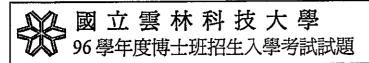
Please express it in polar coordinate  $u(r, \theta, t)$ , with

$$r = \sqrt{x^2 + y^2}$$
 and  $\theta = Tan^{-1}(y/x)$ 

## 2. (30%)

The position of a moving particle is given by  $\vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}$ .

Please find the tangent and normal components of the acceleration vector at any time t, and find the curvature



所別:產業精密機械研究所

科目:工程數學

3. (15%) Consider the initial value problem

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0, \quad x(0) = 1, \quad x'(0) = c,$$

in which c is a constant. Please determine the value of c such that  $x(t) = e^{-t}$  for all  $t \ge 0$ .

4. (15%) Consider the initial value problem

$$\frac{dx}{dt} + x = t^{10}, \quad x(0) = 10.$$

Please find a function f(t) such that  $x(t) = f(t) + \int_0^t e^{-\tau} (t - \tau)^{10} d\tau$ .

- 5. (10%) Please expand  $f(t) = \sin(t)$ ,  $-2\pi < t < 2\pi$ , in a complex Fourier series.
- 6. (10%) Please plot the amplitude spectrum of the periodic wave that is the periodic extension of the function  $f(t) = \sin(t)$ ,  $-\pi < t < \pi$ .