國立雲林科技大學
97學年度博士班招生考試試題

所別：產業精密機械研究所
科目：工程數學 (4)

Prob. 1. (25%)

A line in 3-dimensionl space $R^3$ is represented by a position vector $\vec{p}$ in $R^3$ and given as $\vec{p}=t\vec{v}$, where $t \in R$ and $\vec{v} \in R^3$.

A linear transformation defined by the projection of $\vec{x}$ onto the line given above.

(1) Please define the linear transformation in vector form.

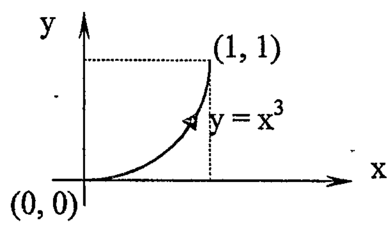
(2) Please find the eigenvalues and the corresponding eigenvectors of the linear transformation.

Prob. 2. (25%)

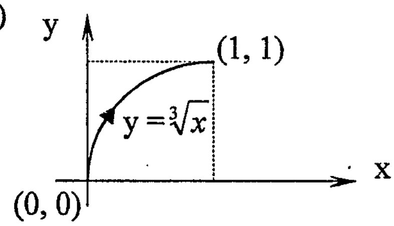
Given $\vec{F}(t)=-2y\,\vec{i}+(5y-2x)\vec{j}$ and $\vec{r}(t)=x\,\vec{i}+y\,\vec{j}$,

please compute the line integral $\int_C \vec{F}\cdot d\vec{r}$ with C as the following paths.

(a)




(b)




(c) Please show the reason why they have the same results.

(Hint: **<u>Try to find the potential function</u>** $\phi(x,y)$ which $\vec{F}$ is derived from.)

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3. Solve the initial value problem $\left(e^{2y} - y\cos xy\right)dx + \left(2xe^{2y} - x\cos xy + 2y\right)dy = 0$ subject to $y(0) = 1$. (15%)

4. Let $\delta(\cdot)$ denote the Dirac delta function. Solve $y'' + 4y = \delta(t-1)$ subject to $y(0) = 0$ and $y'(0) = 0$. (15%)

5. Define $f(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 \le t < \pi \end{cases}$; $f(t + 2\pi) = f(t)$. Solve $y'' + 2y = f(t)$ subject to $y(0) = 0$ and $y'(0) = 0$. (20%)

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科目：工程數學

1. (20%)
A two-dimension wave equation in Cartesian coordinate $u(x,y,t)$ is shown below

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \text{ where c is constant parameter.}$$

Please express it in polar coordinate $u(r,\theta,t)$, with

$r = \sqrt{x^2 + y^2}$ and $\theta = Tan^{-1}(y/x)$

2. (30%)
The position of a moving particle is given by $\vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} + \frac{1}{3}t^3\vec{k}$.

Please find the tangent and normal components of the acceleration vector at any time t, and find the curvature

3. (15%) Consider the initial value problem

$$\frac{d^2x}{dt^2}+3\frac{dx}{dt}+2x=0,\ \ x(0)=1,\ \ x'(0)=c,$$

in which $c$ is a constant. Please determine the value of $c$ such that $x(t)=e^{-t}$ for all $t \geq 0$.

4. (15%) Consider the initial value problem

$$\frac{dx}{dt}+x=t^{10},\ \ x(0)=10.$$

Please find a function $f(t)$ such that $x(t)=f(t)+\int_0^t e^{-\tau}(t-\tau)^{10}d\tau$.

5. (10%) Please expand $f(t)=\sin(t),\ \ -2\pi<t<2\pi,$ in a complex Fourier series.

6. (10%) Please plot the amplitude spectrum of the periodic wave that is the periodic extension of the function $f(t)=\sin(t),\ \ -\pi<t<\pi$.