國立雲林科技大學
九十學年度研究所碩士班入學考試試題

系所：電資所
科目：電子電路

計算題共六題

1. (20%)An amplifier with an input resistance of 10kΩ, when driven by a current source of 1μA and a source resistance of 100kΩ, has a short-circuit output current of 10mA and an open-circuit output voltage of 10V. When driving a 4-KΩ load, what are the voltage gain, current gain, and power gain expressed as ratios and in dB?

2. (20%)In the circuit of Fig. 1, the NMOS transistor has $|V_t|$=0.9V and V_A=50V, and operates with V_D=2V.
   a. What is the voltage gain v_o/v_i?(10%)
   b. What do V_D and the gain become for I increased to 1mA?(10%)

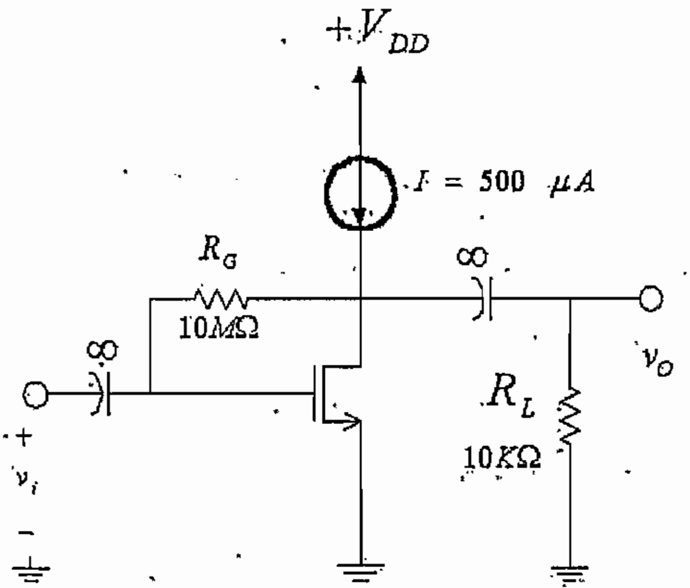

Fig. 1

3. (10%)If in the circuit of Fig. 2, A is an ideal voltage amplifier of gain 100V/V, find midband gain A_M, frequency response in the low-frequency band $F_L(s)$, and frequency response in the high-frequency band $F_H(s)$.

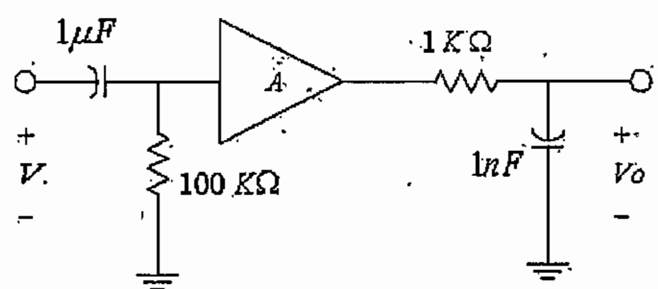

Fig. 2

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系所：電資所
科目：電子電路

4. (25%) For the amplifier shown in Fig. 3, assuming that V_S has a zero dc component and the two BJTs have current gain $\beta = 100$.
   a. Determine the type of the corresponding feedback topology. (5%)
   b. Find the dc emitter currents of Q_1 and Q_2. (8%)
   c. Use feedback analysis to find V_O/V_S and Rin. (12%)

5. (10%) The BiCMOS follower shown in Fig. 4 uses devices for which $V_{BE} = 0.7$ V and $V_{CEsat} = 0.3$ V, $\mu_n C_{ox} W/L = 20\ mA/V^2$, and $V_t = -2$V. (Neglect the channel-length modulation and body effect for the depletion NMOS transistor Q_2.) For linear operation, please answer the following questions.
   a. What is the range of output voltages obtained with a load resistor $R_L = 100\ \Omega$? (5%)
   b. Determine the corresponding power-conversion efficiency for the smallest load resistor allowed for which a 1-V peak sine-wave output is available. (5%)

6. (15%) Design a Butterworth filter that meets the following low-pass specifications: the frequency of passband edge $f_P = 10$ kHz, the maximum allowed variation in passband transmission $A_{max} = 2$ dB, the frequency of stopband edge $f_S = 15$ kHz, and the minimum required stopband attenuation $A_{min} = 15$ dB.
   a. Find the minimum value of N such that the Nth-order Butterworth satisfies the desired specifications. (5%)
   b. Determine the corresponding transfer function $T(s)$ in (a). (10%)

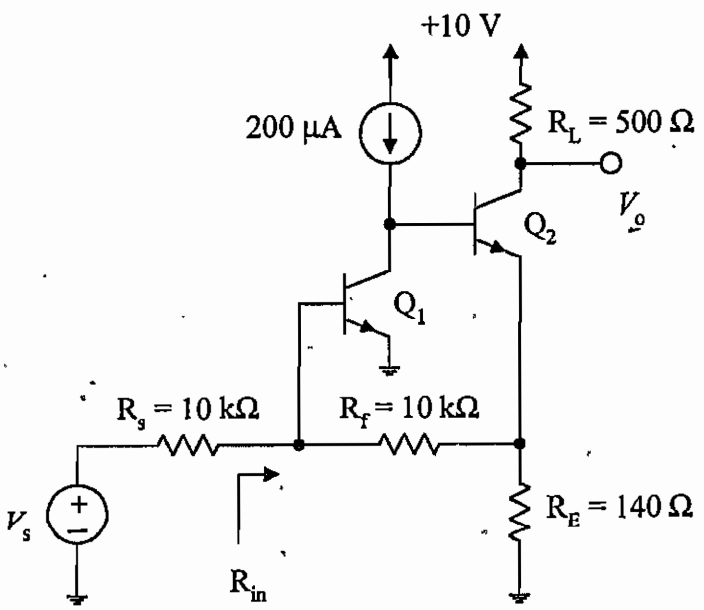

Fig. 3

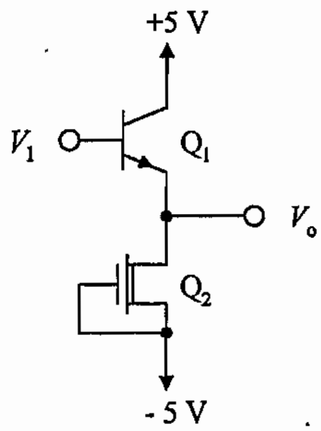

Fig. 4

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系所：電資所
科目：計算機系統

1. Table 1 shows the Booth's Algorithm for the multiplication operation ($b \times a$), where a and b are presented by "$a_n\ a_{n-1}\ \ldots\ a_i\ a_{i-1}\ \ldots\ a_0$" and "$b_n\ b_{n-1}\ \ldots\ b_i\ b_{i-1}\ \ldots\ b_0$".

Table 1

a_i	a_{i-1}	Operation
0	0	Do nothing
0	1	Add b
1	0	Subtract b
1	1	Do nothing

Table 2

Current bits		Previous bit	Operation	Reason
a_{i+1}	a_i	a_{i-1}		
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

(a) Please fill in the Table 2 to revise the Booth's algorithm to look at 3 bits at a time and compute the product 2 bits at a time. (15%)

(b) Assume b is "0 1 0 1 0 1" and a is "0 1 1 0 1 1". Please use Table 2 results to do the $a \times b$ operation. (5%)

2. If we design a CPU that supports eight instructions. Each instruction may be implemented by followed operations: instruction fetch, register read, ALU operation, data access, and register write, as following table shown.

Instruction class	Instruction fetch	Register read	ALU operation	Data access	Register write	Total time
Load (lw)	2 ns	1 ns	2 ns	2 ns	1 ns	8 ns
Store (sw)	2 ns	1 ns	2 ns	2 ns		7 ns
ALU-type (add, sub, and, or, slt)	2 ns	1 ns	2 ns		1 ns	6 ns
Branch (beq)	2 ns	1 ns	2 ns			5 ns

Assume a program that need execute the following instruction mix: 20 loads, 20 stores, 50 ALU-type, and 10 branches. According to the following condition, please find the CPI, execution time, and minimum clock cycle time.

(a) An implementation in which every instruction operations in single clock cycle of a fixed length. (5%)

(b) An implementation where every instruction executes in single clock cycle using a variable-length clock, which for each instruction is only as long as it needs to be. (5%)

(c) Using 5-stage (instruction fetch, register read, ALU operation, data access, register write) pipeline to implement this CPU. (5%)

3. In the pipeline technology, what are the structural hazards, control hazards, and data hazards? (15%)

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4. Explain how the disabling and enabling of interrupts is useful in implementing mutual exclusion primitives on uniprocessor systems. (15%)

5. A process is known to be generating a large number of page faults. Argue both for and against giving the process high priority for the CPU. (15%)

6. A program is made of two control sections as shown below. Suppose that these two control sections are assembled independently of each other.
   (a) List the symbol tables of the two control sections which are established during pass 1 of a two pass assembler. (12%)
   (b) List the external symbol table ESTAB which is established during pass1 of a two pass linking loader. It is assumed that the starting address is 4000H where the program is loaded. (8%)

```
Relative
Location
0000      AA    START   0
                EXTDEF  A1
                EXTREF  B1, BB
0000      AGO   LDA     A2
0003           +STA     B1
0007            LDA     A4
000A           +JSUB    BB
000E      A1    WORD    32
          A2    EQU     A1+3-AA
          A3    EQU     16
0011      A4    WORD    A3+A1
                END     AGO


0000      BB    START   0
                EXTDEF  B1
                EXTREF  A1
0000      BGO  +STA     A1
0004      B1    WORD    16
                END
```

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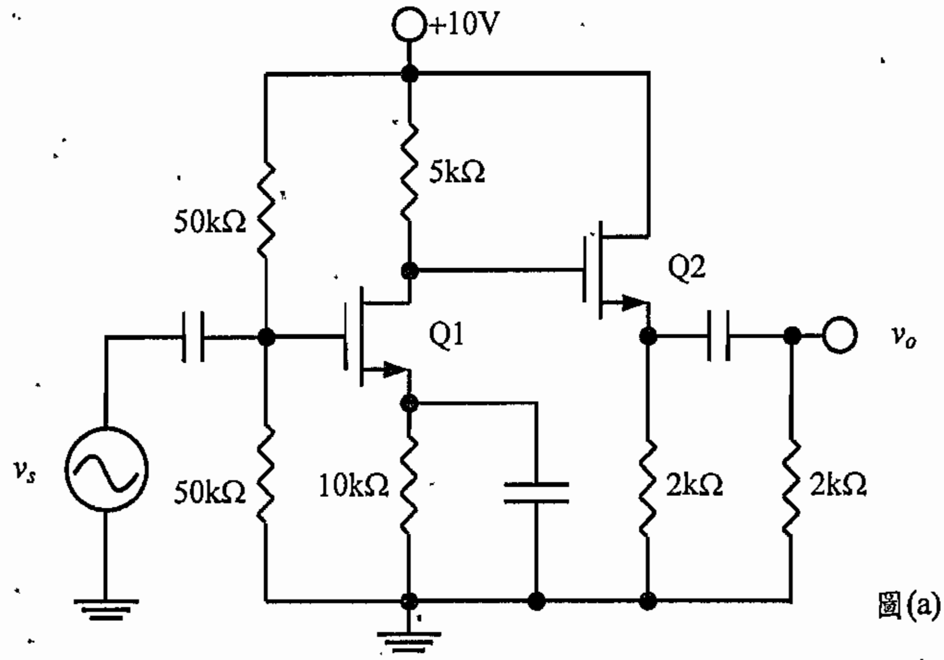
系所：電資所
科目：微電子學

1. Define or explain the following terms:
(a) The Mass-Action Law of semiconductor **(5%)**
(b) Donor impurities for silicon **(5%)**
(c) Schottky-barrier junction **(5%)**

2. Describe the effects of the diffusion capacitance and the transition capacitance of a *p-n* junction diode under forward and reverse bias conditions, respectively. **(20%)**

3. Sketch the typical common-emitter output characteristics of an *npn* bipolar junction transistor and explain the Early effect of the transistor. **(15%)**

4. 圖(a)電路中的電晶體為性能相同 MOSFET，其 $k_n = 0.25\ mA/V^2$ ($= 0.5 \cdot \mu_n \cdot C_{ox} \cdot W/L$)，$V_{Th} = 1.5\ V$，$\lambda = 0$，試計算此電路的電壓增益(A_v)為何？ **(15%)**



圖(a)

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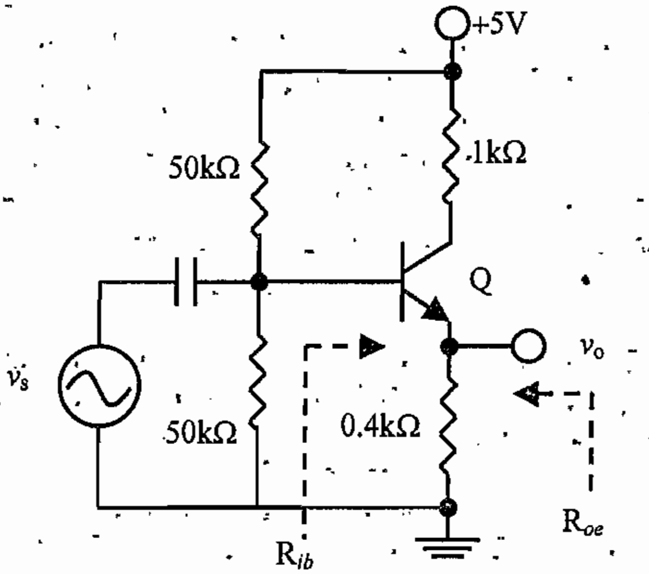
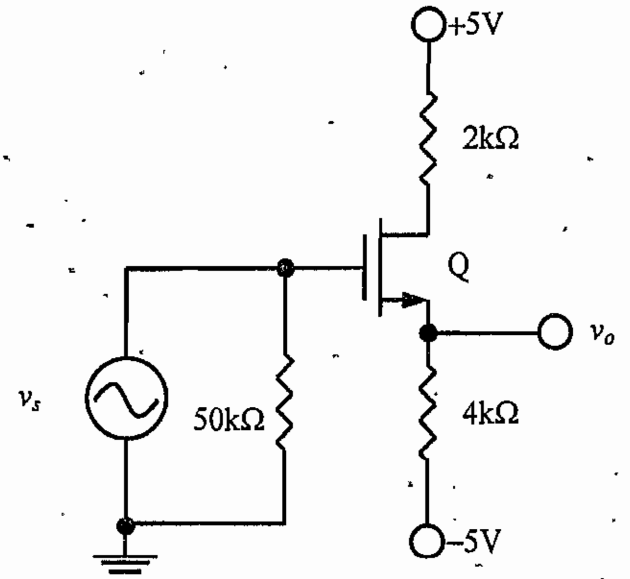
系所：電資所
科目：微電子學

5. 圖(b)放大電路，其中電晶體的 $V_{BE(on)} = 0.7$ V，β 值為 100，V_A 為 150 V，試計算此放大電路的輸入阻抗 (R_{ib}) 及輸出阻抗 (R_{oe})及電壓增益(A_v)為何？(熱電位 = 26mV) **(15%)**



圖(b)

6. 圖(c)所示電路，電晶體之參數為 $k_n = 0.4$ mA/V^2 (= 0.5 • μ_n • C_{ox} • W/L)，V_{Th} = 2.0 V，λ = 0.01/V，C_{gs} = 1 pF 且 C_{gd} = 1 pF，試計算此電路的電壓增益(A_v)及高 3dB 頻率(f_β)為何？ **(20%)**



圖(c)

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系所：電資所
科目：半導體元件

1. In a silicon PN junction, the doping density is linearly graded and symmetrical from −W/2 to +W/2, where N_D-N_A = Bx. Derive the expression for the junction capacitance. (20%)

2. An N channel GaAs MESFET has Φ_B = 0.88 V, $N_D = 10^{17}$ cm^{-3}, a = 0.25 μm, L = 1 μm and Z = 10μm. Given μ_n = 5000 cm^2/V-s and ε_r = 13.1, determine (a) the pinchoff voltage, (b) the threshold voltage and the type of device, (c) the drain voltage at saturation for V_G = 0, and (d) the drain current at saturation for V_G = 0. (20%)

3. Consider a non-ideal effect on Schottky barrier height due to image-force-induced lowering of the potential barrier. The image effect is shown in Figure 1. Assume the semiconductor is n-type. (a) Write down the expression of the force F on the electron due to the coulomb attraction with the image charge. (b) Derive the expression of the potential $\phi(x)$ assuming that no other electric fields exist. (c) Write down the expression of the potential $\phi(x)$ assuming that an electric field $-E$ (E > 0) is present in the semiconductor depletion region. (d) Derive the Schottky barrier lowering $\Delta\phi$. (20%)

4. Consider a uniformly doped silicon npn bipolar transistor at T = 300 K with a base doping of $N_B = 1\times10^{17}$ cm^{-3} and a collector doping of $N_C = 5\times10^{15}$ cm^{-3}. Assume the metallurgical base width is 0.6 μm, D_B = 25 cm^2/sec, and V_{BE} = 0.6 volt. (a) Calculate the neutral base width x_B for V_{CB} = 2 volts and 10 volts. Neglect the B-E space charge region. (b) Write down the expression for excess electron concentration $\delta n_B(x)$ in the base. Assume that $x_B << L_B$. (c) Calculate J_C when V_{CB} = 2 volts. (d) Calculate the Early voltage of the transistor. (20%)

5. Consider a MOS system with an n$^+$ polysilicon gate and a p-type silicon substrate doped to $N_a = 5\times10^{16}$ cm^{-3}. Assume $Q'_{ss} = 1.5\times10^{11}$ cm^{-2}. (a) From Figure 2, find the work function difference ϕ_{ms}. (b) Calculate the maximum space charge width at the surface of the substrate. (c) Determine the oxide thickness such that the threshold voltage = 0.5 volt. (d) Draw the energy-band diagram through the MOS structure at the threshold inversion point. (20%)

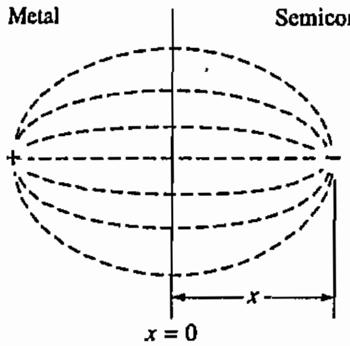

Figure 1 Image effect for Problem 3.

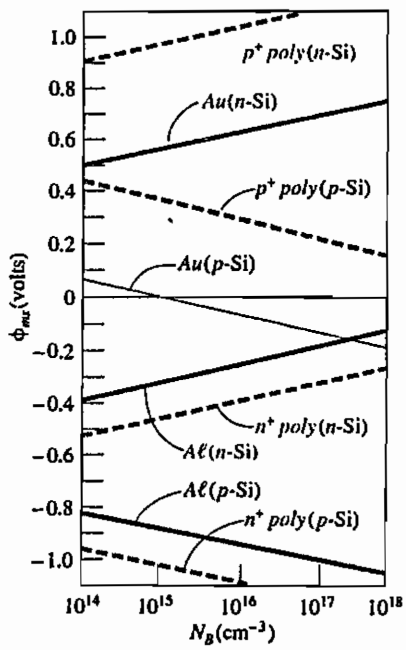

Figure 2 Metal-semiconductor work function difference for Problem 5.

B.4 Silicon, gallium arsenide, and germanium properties (T = 300°K)

Property	Si	GaAs	Ge
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Crystal structure	Diamond	Zincblende	Diamond
Density (g/cm^{-3})	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ, (volts)	4.01	4.07	4.13
Effective density of states in conduction band, N_c, (cm^{-3})	2.8×10^{19}	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band, N_v, (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass, $\left(\frac{m^*}{m_0}\right)$			
Electrons	$m_l^* = 0.98$ $m_t^* = 0.19$	0.067	1.64 0.082
Holes	$m_{lh}^* = 0.16$ $m_{hh}^* = 0.49$	0.082 0.45	0.044 0.28
Effective mass (density of states)			
Electrons, $\left(\frac{m_n^*}{m_0}\right)$	1.08	0.067	0.55
Holes, $\left(\frac{m_p^*}{m_0}\right)$	0.56	0.48	0.37

B.3 Physical constants

Avogadro's number	$N_A = 6.02 \times 10^{+23}$ atoms per gram molecular weight
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$e = 1.60 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s $\frac{h}{2\pi} = \hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.998 \times 10^{10}$ cm/s
Thermal voltage (T = 300°K)	$V_t = \frac{kT}{e} = 0.0259$ volt $kT = 0.0259$ eV

B.6 Properties of SiO_2 and Si_3N_4 (T = 300°K)

Property	SiO_2	Si_3N_4
Crystal structure	[Amorphous for most integrated circuit applications]	
Atomic or molecular density (cm^{-3})	2.2×10^{22}	1.48×10^{22}
Density ($g\text{-}cm^{-3}$)	2.2	3.4
Energy gap	≈9 eV	4.7 eV
Dielectric constant	3.9	7.5
Melting point (°C)	≈1700	≈1900

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系所：電資所
科目：計算機概論 (甲)

1. In Figure 1, there is a block diagram for a binary divider to divide an 7-bit number (x7~x1) by a 2-bit number (y1~y2). The division can be carried out by a series of subtract and shift operation. The right 5 bits (x5~x1) of the dividend register should be used to store the quotient. The system has an input ST that starts the division process.

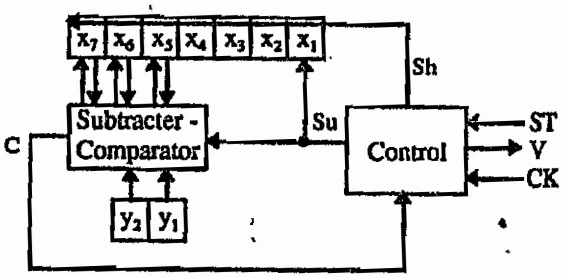

Figure 1

a. Draw a logic diagram for the Dividend register and subtracter-comparator; (8pts)

b. Draw the state graph for the control circuit; (8pts)

c. Give the contents of the dividend register and value of C at each time step if initially the dividend is 0111001 and divisor is 11. (7pts)

Time step	Dividend register X	C
t0	0111001	0
t1		
:		
:		

2. a. Find all of the static hazards in Figure 2. For each hazard, specify the values of the variables which are constant and the variable which is changing. (6pts)

b. Indicate how all of these hazards could be eliminated by adding gates to the existing networks. (You cannot change any of the connections in the given networks.) (8pts)

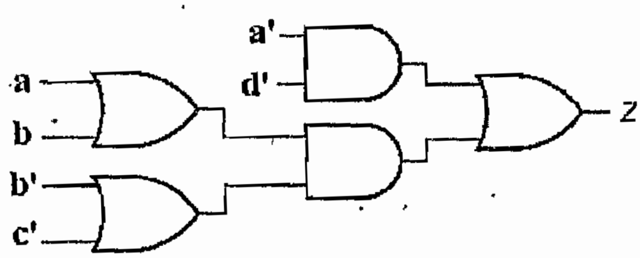

Figure 2

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科目：計算機概論 (甲)

3. a. A sequential network has an input X and an output Z. The output is Z=1iff the total number of 1's received is divisible by 4. (Note: 0, 4, 8, 12, ... are divisible by 4). Find a Mealy state graph and table for the network. (7pts)

b. Reduce the following state table to a minimum number of states. (6pts)

Present	Next State		Present Output	
State	X = 0	1	X =0	1
a	c	f	0	0
b	d	e	0	0
c	h	g	0	0
d	b	g	0	0
e	e	b	0	1
f	f	a	0	1
g	c	g	0	1
h	c	f	0	0

4. For the graph given in Figure 3, the nodes are labeled from 1 to 6 and the arcs are labeled with the distance between two nodes. (20pts)

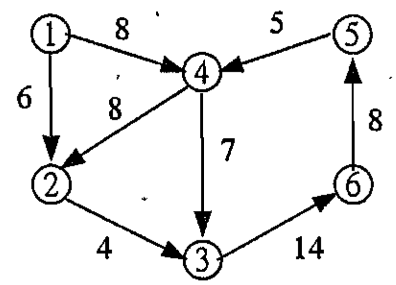

Figure 3

a. What is an elementary loop? How many are there in the graph? (3 pts)

b. What is an adjacency matrix? Please write down the adjacency matrix of the graph to represent the interconnections among nodes. (3 pts)

c. What is the transitive closure of the graph? (4 pts)

d. Please define a data structure, in whatever language you feel comfortable, suitable to represent the graph? (4 pts)

e. Given nodes *i* and *j*, Please describe an algorithm to calculate the shortest path between them. (6 pts)

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5. For an expression tree as shown in Figure 4 (20pts)

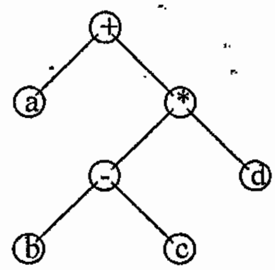

Figure 4

a. please write down the infix, prefix, postfix expressions of the tree (6 pts)
b. please write down a recursive procedure to travse the tree in prerorder (6 pts)
c. a postfix expression can be evaluated by a stack machine, i.e. you push the operands into the stack and pop them out for evaluation. Please draw the block digram of a stack machine to evaluate the postfix expression. (8 pts)

6. Floating point calculation (10 pts).

a. Please describe the format of a 32-bit floating point number (4 pts)
b. Please write down the procedures in performing a floating point addition by a floating point processor. Note that the resultant floating point number should be normalized. (6 pts)

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九十學年度研究所碩士班入學考試試題

系所：電資所
科目：計算機概論 (乙)

1. From the list: 26, 39, 104, 195, 403, 504, 793, 995, 1156, 1673, extract a collection of numbers whose sum is 3165. How effective is your approach to the problem? (15 points)

2. Draw a flowchart representing the structure expressed by the following C, C++, and Java statement. (10 points)

```
switch (pigment)
    { case "red" :   platte = 1;
      case "green" :  platte = 2;
      case "blue" :  platte = 3;
    }
```

3. List five resources to which a multitasking operating system might have to coordinate access. (10 points)

4. Suppose $f(n)$ is a function call with the value of $f(n)$ is n. The running time of $f(n)$ is $O(n^2)$. Consider the procedure whose body is

```
sum := 0;
for i := 0 to f(n) do
    sum := sum + i;
```

   What is the running time of this procedure? (15 points)

5. Use a finite state transition machine to extract tokens from a string representing an infix expression. For example, if the string is "69-(25+20)/15", then the tokens extracted are "69", "-", "(", "25", "+", "20", ")", "/", and "15". Write an algorithm that uses the finite state transition diagram as a data structure and that extracts the tokens from a string of infix expression. The arithmetic operators are +, -, *, and /. Use the above example to illustrate the transition of states during the token extraction process. (15 points)

6. Consider the grammar
   - S → AB | aaB
   - A → a | Aa
   - B → b

   (a) Show that the grammar is ambiguous. (3 points)
   (b) Give five example strings generated by the grammar. (3 points)
   (d) Construct an unambiguous grammar equivalent to the grammar. (4 points)

7. What is a heap? Write an algorithm to build a heap whose inputs are unordered numbers. Illustrate your algorithm with the example, 11, 3, 8, 6, 46, 26, 31, 40, 25, 21. What is the time complexity of your algorithm? (15 points)

8. What is the definition of Fibonacci numbers? Write an algorithm to compute the nth Fibonacci number. What is the time complexity of your algorithm? If the complexity is not linear, please write another algorithm with linear time complexity. (10 points)

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系所：電資所
科目：通信系統

1. (13%) Determine the convolution of $x(t) = t^2u(t)$ and $h(t) = e^{3t}u(t)$, where $u(t)$ is the unit step function.

2. (13%) Let $X_1, X_2, \ldots, X_n$ denote independent and identically distributed random variables, each with p.d.f. $f_X(x)$. If $Y = \max\{X_1, X_2, \ldots, X_n\}$, find the p.d.f. of Y.

3. (14%) A Markov process is a process with one step memory, i.e., a process such that

$$p(x_n|x_{n-1}, x_{n-2}, x_{n-3}, \ldots) = p(x_n|x_{n-1})$$

for all n. Show that for a stationary Markov process the entropy rate is given by $H(X_n|X_{n-1})$.

4. The input-output relation of a system is given by $y(t) = x(at-1) + y(t-1)$, where $a \neq 0$.
   (a) (5%) Find the range of a such that the system is causal.
   (b) (5%) Find the range of a such that the system is time-varying.
   (c) (5%) Is the system linear? Explain.
   (d) (5%) Let the Fourier transforms of $x(t)$ and $y(t)$ be $X(f)$ and $Y(f)$, respectively. Express $Y(f)$ in terms of $X(f)$.

5. An ternary digital modulator generates three signals

$$s_1(t) = 2g_1(t) + g_2(t), \quad s_2(t) = g_2(t) + g_3(t), \quad s_3(t) = g_1(t) + g_3(t),$$

with the same probability, where $\{g_k(t),\ k = 1,2,3,\ 0 \le t \le T\}$ forms an orthonormal basis. The received signal $r(t)$ at the input of the receiver is corrupted by a zero-mean white Gaussian noise $n(t)$ with variance σ^2:

$$r(t) = s_m(t) + n(t), \quad m \in \{1,2,3\}.$$

   (a) (5%) Find the signal constellation of $s_m(t)$, $m = 1,2,3$.
   (b) (5%) Design the optimum (MAP) coherent receiver with reference signals $\{s_1(t), s_2(t), s_3(t)\}$.
   (c) (5%) Suppose a new set of signals $\{x_m(t),\ m = 1,2,3\}$ is constructed by

$$x_m(t) = s_m(t) - f(t), \quad m = 1,2,3,$$

   where $f(t) = f_1g_1(t) + f_2g_2(t) + f_3g_3(t)$. Find $f(t)$ that minimizes the average energy of $x_1(t), x_2(t)$ and $x_3(t)$.
   (d) (5%) Design the optimum coherent receiver with reference signals $\{x_1(t), x_2(t), x_3(t)\}$.
   (e) (5%) Which of the two receivers in Parts (c) and (e) has a lower average error probability? Explain.

6. Suppose we know that $x^4 + x^3 + x^2 + 1$ and $x^5 + x^2 + x + 1$ are two valid codewords of a (7,4) cyclic code.
   (a) (5%) Find the generator polynomial $g(x)$=?
   (b) (5%) Find the corresponding generator matrix $G = [I_4|P]$=?
   (c) (5%) Is this (7,4) cyclic code a linear block code? Explain.

國立雲林科技大學
九十學年度研究所碩士班入學考試試題

系所：電資所
科目：電磁學

說明：本試卷共六大題，總分共計１００分。

（20分）1. A point charge q is placed at a point A, as shown in Fig.1, which is at a distance d from the separation boundary of two semi-infinite homogeneous dielectrics whose permittivities are ε_1 and ε_2. Find the potentials φ_1 and φ_2 by the method of images. Hint: The solution should be sought in the form of the equations (1) and (2), where r_1 and r_2 are the distance.

$$\varphi_1 = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r_1} - \frac{q'}{r_2}\right), Z \geq 0 \quad \text{................(1)}$$

$$\varphi_2 = \frac{1}{4\pi\varepsilon_o}\frac{q''}{r_1}, Z < 0 \quad \text{.........................(2)}$$

（15分）2. The insulator between the inner conducting cylinder a and outer cylindrical shell b is composed of four sectors of equal dimensions but different dielectric constants, ε_1, ε_2, ε_3 and ε_4, see Fig.2. The radius of cylinder a is r_a and the inner radius of outer cylindrical shell b is r_b. In case the cylinders extend indefinitely, find the capacitance per unit length.

（20分）3. An infinitely long solenoid with air core having a radius b and n closely wound turns per unit length, as shown in Fig.3. The windings are slanted at an angle α and carry a current I. Determine the magnetic flux density both inside and outside the solenoid.

（10分）4. A light ray is incident from air obliquely on a transparent sheet of thickness d with an index of refraction n, as shown in Figure.4. The angle of incidence is θ_i. Find (a) the distance l_1 at the point of exit, and (b) the amount of the lateral displacement l_2 of the emerging ray.

（15分）5. Determine the electric field intensity at the center of a small spherical cavity cut out of a large block of dielectric in which a polarization $\mathbf{P} = \mathbf{a}_z P$ exists, as shown in Fig.5.

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（20分）6. Explain the following terms:

(a) Eddy currents
(b) Brewster angle
(c) Hysteresis loop
(d) Uniqueness theorem of electrostatics
(e) Helmholtz's theorem

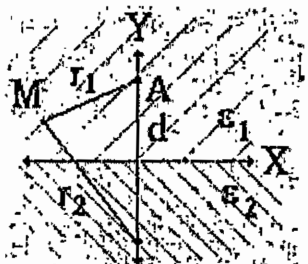

Figure 1

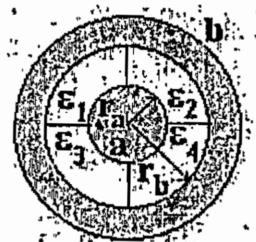

Figure 2

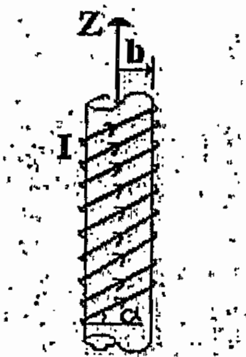

Figure 3

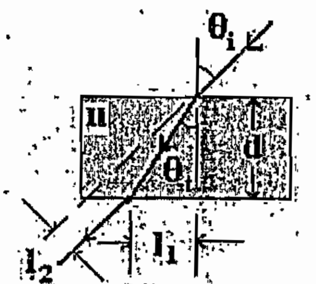

Figure 4

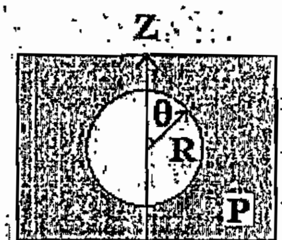

Figure 5

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系所：電資所
科目：線性代數

1. (a) (5%) What is the LU factorization of a matrix?
   (b) (10%) Perform the LU factorization of the matrix

$$A = \begin{bmatrix} 8 & 12 & -4 \\ 6 & 5 & 7 \\ 2 & 1 & 6 \end{bmatrix}$$

2. (a) (5%) What is the QR factorization of a matrix?
   (b) (10%) Perform the QR factorization of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

3. (a) (5%) What is the singular value decomposition (SVD) of a matrix?
   (b) (15%) Perform the SVD of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

4. Suppose $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$.
   (a) (8%) Is A diagonalizable? Explain.
   (b) (8%) Derive a general expression for A^n.

5. Consider the matrix

$$A = \begin{bmatrix} -1 & 3 & 0 \\ 0 & 2 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

   (a) (4%) Compute A^{-1}.
   (b) (4%) Express A^{-1} in terms of I, A and A^2. (Hint: Use Caley-Hamilton Theorem.)
   (c) (8%) Find a nonsingular matrix C with initial entry $c_{1,1} = 1$ that transforms A to the following Jordan canonical form:

$$C^{-1}AC = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

6. Answer the following problems on orthogonalization and vector space.
   (a) (6%) Describe the Gram-Schmidt process, using k vectors $x_1, \ldots, x_k$.
   (b) (6%) Find an orthonormal basis for the subspace of R^3 spanned by three vectors $x_1 = (1,1,0)$, $x_2 = (1,0,1)$, and $x_3 = (3,1,2)$.
   (c) (6%) Find the null space in Part (b).